

The spread of hierarchical cycles

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We propose a relational method to study impredicative systems, which are natural systems that have models containing hierarchical cycles. The method is formulated in category theory in terms of alternate descriptions and their functorial connections. This general theory of impredicative systems has implications in the biological, psychological, and social realms, from which we offer many exemplifications.

Keywords: hierarchical cycles; impredicative systems; alternate descriptions; relational methods; closure to efficient causation; clef systems

1. Introduction

During the past 50 years, the idea of connections linking systems and their components, generating cycles that tie together components and systems in such a way that the fragmentation of the system always implies loss of information, has been frequently advanced. To mention only some authors, Bateson, Capra, Hofstadter, Luhmann, Maturana, Rosen, and Varela are advocates of this idea. These component–system connections form what we shall call 'hierarchical cycles'. When components pertaining to a hierarchical cycle are separated from their system, they behave differently (and *may* have a different nature) from the way in which those same components behave within their system.

Hierarchical cycles must be carefully distinguished from sequential (i.e. 'horizontal') cycles. The latter are well represented by feedback and autocatalytic loops, where elements of the same kind interact with each other. Nonlinear phenomena mostly rely on sequential cycles.

Unfortunately, the above-mentioned scholars – with the remarkable exception of Robert Rosen (in his subject known as *relational biology*) – do not usually distinguish as sharply as necessary between sequential and hierarchical cycles. This unfortunate state of affairs – quite typical, however, of newborn, still unfolding ideas – has contributed to obscuring the scientific importance of hierarchical cycles.

Hierarchical cycles represent a substantial move towards a relational understanding of systems. According to this perspective, many natural systems are relational systems *over* a material basis. Nobody denies that an underlying material basis is needed. The real nature of these natural systems, however, is not conveyed by their material basis. The claim implies that living, psychological, and social systems are not *properly* understandable by studying the 'materials' that happen to bear them or the physical environment in which they happen to be embedded. This is not to deny that *some* information may derive from

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their material bases. The thesis instead claims that what is *specifically living*, *psychological or social* of living, psychological, or social systems does not derive from their underlying supporting bases.

One of the intriguing aspects of a system characterized by hierarchical cycles is that the elements generating the system's dynamical continuity *may* be different from the elements composing the system's material basis. We shall come back to this issue below.

Organisms, minds, and societies are systems able to outlive their elements – new elements are born, others die off, yet others move from one system to another. All these modifications notwithstanding, these types of systems show some kind of *stability* which, for the most part, is independent of the continuous transformation of the set of their constituent elements.

The presence of hierarchical cycles dramatically constrains the modelling of the relevant system. To mention but one single result, a system containing a hierarchical cycle must have a non-simulable model, which implies that no simulable description of that system will ever be complete. This result does not imply that there can be no model of hierarchical cycles at all. There are plenty of useful algorithmic models, just with the caveat that these will be, by definition, *incomplete*. They may nevertheless be fruitful endeavours. One learns a tremendous amount even from partial descriptions.

This paper presents a few exemplifications of hierarchical cycles (aka impredicative systems or self-referential systems), showing that they can be treated in a uniform way. We shall collect data from the various scientific fields in which the idea of hierarchical cycle has been proposed, namely, biology and cognitive and social sciences.

Other theoretical perspectives resembling the present proposal have been recently advanced. Notable examples are memory evolutive systems by Ehresmann and Vanbremeersch (2007) and the development of supercategories and higher order types of complexity by Baianu *et al.* (2011). Both of them essentially rely on the power of category theory and both of them develop, albeit in different ways, the idea of iterative constructions of systems over systems in which the systems at different layers present specific properties. In our approach, we emphasize the graph-theoretic aspects of category theory, with the advantage that it is more visual than strictly abstract-algebraic methods.

2. Basic elements

In this section, we review some of the key concepts of relational biology to make this paper (more or less) self-contained. For further details, the reader is invited to read the book *More Than Life Itself: A Synthetic Continuation in Relational Biology* (Louie 2009).

2.1 Category theory

Category theory is a useful metalanguage in these discussions. For a simple and concise introduction, one may consult the Appendix in Louie (2009).

A *category* consists of a collection of *objects*, and for each pair of objects A and B a *hom-set* denoted by H(A, B). A member of a hom-set is called a *morphism*. The general framework assumes very little about the morphisms; they only need to be closed under composition and include the identity morphisms. The category axioms are usually interpreted within set theory, whence an object is a set with a prescribed structure and a morphism is a mapping preserving this structure.

The category **Set**, in which the objects are sets (without any further requisite structure) and the morphisms are mappings between sets, is the simplest example of a category. A mapping *f* from *A* to *B* may, therefore, also be denoted as $f \in H(A, B)$. Note that the hom-set H(A, B) only needs to represent *a* collection of mappings under consideration

from set A to set B (but, of course, still satisfying the closure requirements in the category axioms), and is not necessarily the collection of *all* mappings from A to B.

2.2 The modelling relation

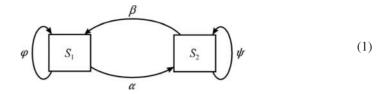


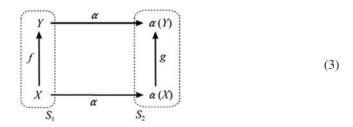
Figure (1) contains the components we need to describe what a *modelling relation* is between a system S_1 and a system S_2 . The crux of the matter lies in the arrows of the diagram, which we have labelled φ , ψ , α , and β . The arrows φ and ψ represent *entailment* in the systems S_1 and S_2 , respectively. The arrow α is called the *encoding* arrow. It serves to associate features of S_1 with their counterparts in S_2 . The arrow β denotes the inverse activity to encoding; namely, the *decoding* of features of S_2 into those of S_1 .

The arrows α and β taken together thus establish a kind of *dictionary*, which allows effective passage from one system to the other and back again. However, we may remark here on the peculiar status of the arrows α and β . Namely, they are not a part of either systems S_1 or S_2 , nor are they entailed by anything either in S_1 or in S_2 .

A modelling relation exists between systems S_1 and S_2 when there is a *congruence* between their entailment structures. The vehicle for establishing a relation of any kind between S_1 and S_2 resides, of course, in the choice of encoding and decoding arrows, the arrows α and β . A necessary condition for congruence involves all four arrows, and may be stated as 'whether one follows "path φ " or "paths α , ψ , β in sequence", one reaches the same destination'. Expressed as composition in mathematical terms, this is

$$\varphi = \beta \circ \psi \circ \alpha. \tag{2}$$

Encoding and decoding maps have certain inherent properties; we shall illustrate using the encoding arrow α . Let $f : X \to Y$ be a mapping representing a process in the entailment structure of the arrow φ in S_1 . Consider a mapping $g : \alpha(X) \to \alpha(Y)$ (which is a process in the entailment structure of the arrow ψ in S_2) that makes the diagram



commute (which means for *every* element *x* in *X*, whether it traces through the mappings *f* followed by α , or through α followed by *g*, one gets the same result in $\alpha(Y)$; i.e. the equality

$$\alpha(f(x)) = g(\alpha(x)) \tag{4}$$

holds for all $x \in X$). Note that this commutativity condition so far places no further restrictions on mapping g itself, other than that it needs to reach the correct final destination.

The mapping g, however, must also *itself* be entailed by the encoding α , i.e.

$$g = \alpha(f),\tag{5}$$

whence the mapping in S_2 is $\alpha(f): \alpha(X) \rightarrow \alpha(Y)$. Then one has the commutative diagram

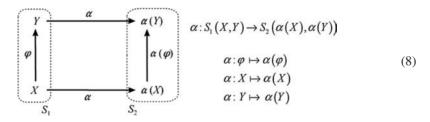
$$\begin{array}{c}
 Y & \alpha & \alpha(Y) \\
 f & \alpha(f) & \alpha(f) \\
 x & \alpha(X) & \alpha(X) \\
 S_1 & S_2
\end{array}$$
(6)

and the equality corresponding to (4), for every element x in X, is

$$\alpha(f(x)) = \alpha(f)(\alpha(x)). \tag{7}$$

It is only when this more stringent condition (7) is satisfied that one has a true *modelling* relation between systems S_1 and S_2 . One then says that there is a congruence between their entailment structures, and that S_2 is a *model* of S_1 .

Category theory may be regarded as a general theory of modelling relations. The kind of congruence (7) between entailment structures is defined by the mathematical entity called *functor*. Stated otherwise, encoding is a functor α from category S_1 to category S_2 :



2.3 Relational diagram

A relational diagram in graph-theoretic form is a representation of interconnected processes in a way that emphasizes the different roles played by different components of the mappings. For a simple mapping $f : A \rightarrow B$ (i.e. $f \in H(A, B)$) in its element-chasing version $f : a \mapsto b$ (where $a \in A$ and $b \in B$), its relational diagram may be drawn as a network with three *nodes* and two *directed edges*, i.e. a directed graph (or *digraph* for short):

$$\xrightarrow{f} \qquad a \qquad b \qquad (9)$$

The *hollow-headed arrow* denotes the *flow* from input (material cause) $a \in A$ to output (final cause) $b \in B$, whence the final cause of the mapping may be identified also as the hollow-headed arrow that terminates on the output

$$\longrightarrow b$$
 (10)

The final cause, output of a mapping, is that which is *entailed*.

The *solid-headed arrow* denotes the induction of or constraint upon the flow by the *processor* (efficient cause) f, whence the efficient cause of the mapping may be identified also as the solid-headed arrow that originates from the processor

$$f \longrightarrow$$
 (11)

The formal cause of the mapping may be identified as the *ordered pair* \langle processor, flow \rangle of the two kinds of arrows:

Relational diagrams of mappings may be composed. For example, consider the two mappings $f \in H(A, B)$ and $g \in H(X, A)$: the codomain of g is the domain of f. Thus

$$X \xrightarrow{g} A \xrightarrow{f} B.$$
(13)

Let the element chases be $f : a \mapsto b$ and $g : x \mapsto a$: the final cause of g is the material cause of f. The relational diagrams of the two mappings connect at the common node a as

 $x \xrightarrow{g} b$ (14)

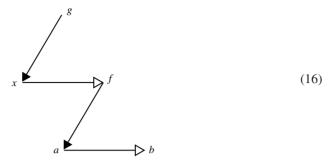
This *sequential composition* of relational diagrams represents the composite mapping $f \circ g \in H(X, B)$ with $f \circ g : x \mapsto b$, and has the abbreviated relational diagram



Note that in this diagram (15) for the single efficient cause $f \circ g$, both efficient causes f and g, as well as the (final) final cause b, are accounted for.

Now consider two mappings $f \in H(A, B)$ and $g \in H(X, H(A, B))$. The mapping g takes an element in X and sends it to an image which is another mapping, one with domain A and codomain B; in short, the codomain of g contains f. Because of this 'containment', the mapping g may be considered to occupy a higher hierarchical level than the mapping f. Let the element chases be $f : a \mapsto b$ and $g : x \mapsto f$: the final cause of g is the efficient cause of f. (In particular, the mapping f is entailed.) Then one has the hierarchical

composition of relational diagrams



A comparison of the two graphs (14) and (16) shows that sequential composition and hierarchical composition are different in kind: they are different both *formally* and in content. Although diagrams (16) may contract into something similar in form to (15), namely



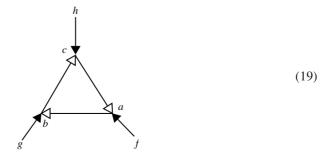
in this abbreviated form, the entailed efficient cause f becomes 'hidden'. Since the accounting (and tracking) of all efficient causes in an entailment system is crucial in our understanding of hierarchical cycles, one needs to preserve every solid-headed arrow. So there will not be any abbreviation of hierarchical compositions.

Sequential cycle 2.4

The mappings in a relational diagram may compose in such a way that a closed path, i.e. a cycle, is formed. (Note that a closed path in the directed graph sense means the arrows involved have a consistent direction.) When the compositions involved in the closed path are all sequential, one has a sequential cycle. This cycle consists of hollow-headed arrows entirely, with peripheral solid-headed arrows. In this cycle, all those entailed are material causes; it is, therefore, a *closed path of material causation*. For example, when three mappings have a cyclic permutation of domains and codomains,

$$f \in H(A,B), \quad g \in H(B,C), \quad h \in H(C,A),$$
(18)

their sequential compositions result in



The three mappings compose to

$$h \circ g \circ f : A \to A, \tag{20}$$

which may, depending on the emphasis, be interpreted as the automorphism

$$a \cong h \circ g \circ f(a), \tag{21}$$

the identity mapping

$$h \circ g \circ f = 1_A \in H(A, A), \tag{22}$$

or the *fixed point a* of the mapping $h \circ g \circ f$,

$$h \circ g \circ f(a) = a. \tag{23}$$

Cyclic permutation of the three mappings also gives

$$f \circ h \circ g : B \longrightarrow B \tag{24}$$

and

$$g \circ f \circ h : C \to C, \tag{25}$$

with the corresponding automorphism, identity mapping, and fixed point interpretations in their appropriate domains.

It is easy to see that the number of mappings involved in a closed path of material causation may be any finite number (instead of three in the example), and the above discussion may be extended accordingly. Thus, a closed path of material causation is formally analogous to the simple relation diagram with a self-loop

(26)

If a relational diagram either contains no closed paths or when the only closed paths are sequential cycles, it is inherently simple, in contrast to those that contain another kind of cycles that is our next topic.

2.5 Hierarchical cycle and its properties

When *two or more* compositions involved in the cycle are hierarchical, one has a *closed path of efficient causation*. (A closed path with exactly one efficient cause is an exceptional case which need not concern us here. Interested readers may consult Section 6.15 in Louie 2009.) In other words, a closed path of efficient causation is an entailment cycle that contains two or more efficient causes. Both the hierarchy of containment and the cycle are essential attributes of this closure.

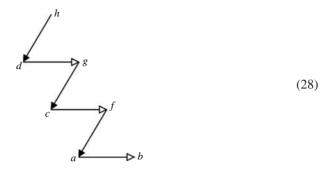
For example, consider three mappings from a hierarchy of hom-sets,

$$f \in H(A,B), \quad g \in H(C,H(A,B)), \quad h \in H(D,H(C,H(A,B))).$$
(27)

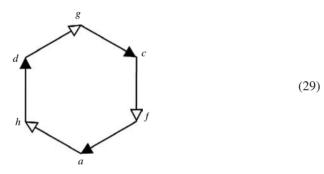


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Their hierarchical compositions form the relational diagram



Now suppose there is a correspondence between the sets *B* and H(D, H(C, H(A, B))) – the many ways to achieve this correspondence are explicated in Louie (2009), in particular, in Chapters 6, 11, and 12. Then, an isomorphic identification between *b* and *h* may be made, and a cycle of hierarchical compositions results:



Formally, we have the following definition.

DEFINITION. A *hierarchical cycle* is the relational diagram in graph-theoretic form of a closed path of efficient causation.

Note that in a hierarchical cycle (for example, arrow diagram (29)), there are *two or more solid-headed arrows* (since a closed path of efficient causation is defined as a cycle containing *two or more* hierarchical compositions). Since a hierarchical cycle is by definition the formal-system representation (i.e. encoding) of a closed path of efficient causation in a natural system, trivially one has the following:

LEMMA. A natural system has a model containing a hierarchical cycle if and only if it has a closed path of efficient causation.

Because of this equivalence of a closed path of efficient causation in a natural system and a hierarchical cycle in its model, the term *hierarchical cycle*, although defined for formal systems, sometimes gets decoded back as an alternate description of the closed path of efficient causation itself. In other words, one may speak of a hierarchical cycle of inferential entailments as well as a hierarchical cycle of causal entailments. Thus, 'hierarchical cycle' joins the ranks of 'set', 'system', etc., as words that inhabit the realms of both natural systems and formal systems.

Just as sequential composition and hierarchical composition are different in kind, so are sequential cycle and hierarchical cycle. Because of this in-kind difference between the two types of cycles, a sequential cycle may also be referred to as a 'horizontal cycle' or a 'flat cycle' to emphasize its non-hierarchical characteristic. A hierarchical cycle has many interesting mathematical properties and, therefore, by extension a natural system N that contains a closed path of efficient causation has the realizations of these properties.

Among the many properties of a natural system N that contains a closed path of efficient causation are the following (See Chapter 7 of Louie 2009 for details.):

1. *N* does not have a *largest model*.

[The largest model (if it exists) is the greatest element in the lattice of models, which implies that every model is its submodel.]

- 2. Not every property of *N* is *fractionable*. [A property of a natural system is fractionable if the natural system can be separated into two parts modelled by disjoint direct summands, such that the property is manifest in one of these parts.]
- There exist models of N that are not *simulable*.
 [A model is simulable if every process is definable by an algorithm.]
- N is an *impredicative* system.
 [Indeed, the containment of a closed path of efficient causation may be used as a definition of *impredicativity* our next topic.]

2.6 Impredicativity

In logic, the *predicate* is what is said or asserted about an object. It can take the role as either a property or a relation between entities. Contrariwise, a definition of an object is said to be *impredicative* if it invokes (mentions or quantifies over) the object itself being defined, or perhaps another set which contains the object being defined. In other words, *impredicativity* is the property of a *self-referencing definition*. (For a formal definition of impredicativity, see Chapter 8 of Louie 2009.)

As an example, consider the definition of *supremum*. Let \leq be a partial order on a set *X* and let $A \subset X$. The subset *A* is *bounded above* if there exists $x \in X$ such that $a \leq x$ for all $a \in A$; such $x \in X$ is called an *upper bound* for *A*. An upper bound *x* for *A* is called the *supremum* for *A* if $x \leq y$ for all upper bounds *y* for *A*. Stated otherwise, $x = \sup A \Leftrightarrow x \leq y$ for all $y \in Y = \{y \in X : y \text{ is an upper bound for } A\}$. Note that the definition invokes the set *Y* and the supremum $x \in Y$, whence the definition of 'supremum' is impredicative.

Impredicative definitions usually cannot be bypassed, and are mostly harmless. But, there are some that lead to paradoxes. The most famous of a problematic impredicative construction is Russell's paradox, which involves the set of all sets that do not contain themselves: $\{x : x \notin x\}$. This foundational difficulty is only avoided by the restriction to a naive set-theoretic universe that explicitly prohibits self-referencing constructions. This implies reducing acceptable mathematical constructions to recursive procedures only – i.e. to algorithmic or rote procedures. There is much in mathematics that goes beyond this severe delimitation.

2.7 Complex and clef systems

Hierarchical cycle is used in the definitions of two important classes of systems: 'complex systems' and 'systems that are closed to efficient causation'.

DEFINITION. A natural system is *complex* if and only if it has a model that contains a hierarchical cycle.

Note that this only requires the existence of a hierarchical cycle that contains two or more processes. There may be many processes in the model that are not part of hierarchical cycles.

DEFINITION. A natural system is *closed to efficient causation* if its every efficient cause is entailed within the system.

In Chapter 6 of Louie (2009), the following two important properties of a natural system are proven to be equivalent:

- (a) its every efficient cause is entailed within the system and
- (b) it has a model that has all its processes contained in hierarchical cycles.

Stated otherwise, in a closed-to-efficient-cause system, *all* processes are involved in hierarchical cycles. Thus, the class of systems that are closed to efficient causation forms a proper subset of the class of complex systems (which are required to have only *some* processes involved in hierarchical cycles). Because of this containment, a closed-to-efficient-cause system may be considered a 'higher order complex system'.

Instead of the verbose 'closed-to-efficient-cause system' or 'systems that are closed to efficient causation', we would like to introduce a new term '*clef* system' (for *cl*osed to *efficient* causation) with the following definition.

DEFINITION. A natural system is *clef* if and only if it has a model that has all its processes contained in hierarchical cycles.

The word 'clef' means 'key'; so this terminology has the added bonus of describing the importance of the class of *clef systems*.

There are different families of these clef systems that are of 'higher order' than complex systems. Three of them will be exemplified in this paper, namely living, psychological, and social systems. Note that in Chapter 11 of Louie (2009), natural systems with a model containing all the processes in hierarchical cycles is shown as the defining property of a *living system*. Since in this paper we are considering also psychological and social systems, we are using the less restrictive new name of *clef system* in its stead.

Although we shall not discuss here in detail whether clef systems are organized into different layers of higher order complexity (e.g. in the way in which Baianu et al. (2011) distinguish between super- and hyper-complexity), at least one basic comment may prove helpful. In fact, the difference between living, psychological, and social systems can be made more explicit by resorting to the theory of levels of reality. The main distinction here is between 'material', 'psychological', and 'social' levels of reality, the ontological categories that are embedded within them and the relations of dependence and independence among the different levels of reality (for details, see Poli 2001, 2006a,b, 2007). Subsequently, the distinction is needed among the various sublevels, such as the physical, chemical, and biological sublevels of the material level, or the economical, political, and legal sublevels of the social level. Leaving apart many details, the only information that is presently needed is the difference between living systems from one side and psychological and social systems from the other side. Living systems pertain to the material level of reality, whereas psychological and social systems are not material systems. The expression 'system over a material basis' acquires then two different specifications: a living system is a system over a physico-chemical basis, whereas mind and society are systems over biological systems, respectively, over the brain and the whole individual organism. This difference contributes to the idea that clef systems, presenting a higher order kind of complexity, may be organized accordingly into different layers of higher order complexity. We leave a more articulated discussion of this issue for another occasion.

3. An early trial

For an early view on hierarchy, here is what von Bertalanffy (1968) wrote:

An Informal Survey of Main Levels in the Hierarchy of Systems. Partly in pursuance of Boulding (1956).

Level	Description and examples	Theory and models
Static structures	Atoms, molecules, crystals, biological structures from the electron-micro- scopic to the macroscopic level	E.g. structural formulas of chemistry; crystallography; anatomical descrip- tions
Clock works	Clocks, conventional machines in general, solar systems	Conventional physics such as laws of mechanics (Newtonian and Einsteinian) and others
Control mechanisms	Thermostat, servomechanisms, homeostatic mechanism in organisms	Cybernetics; feedback and information theory
Open systems	Flame, cells, and organisms in general	 (a) Expansion of physical theory to systems maintaining themselves in flow of matter (metabolism). (b) Information storage in genetic code (DNA). Connection of (a) and (b) presently unclear
Lower organisms	'Plant-like' organisms: increasing differentiation of system (so-called 'division of labour' in the organism); distinction of reproduction and functional individual ('germ track and soma')	Theory and models almost lacking
Animals	Increasing importance of traffic in information (evolution of receptors, nervous systems); learning; beginnings of consciousness	Beginnings in automata theory (S-R relations), feedback (regulatory phenomena), autonomous behaviour (relaxation oscillations), etc.
Man	Symbolism; past and future, self and world, self-awareness, etc., as consequences; communication by language, etc.	Incipient theory of symbolism
Socio-cultural systems	Populations of organisms (humans included) symbol-determined communities (cultures) in man only	Statistical and dynamic laws in population dynamics, sociology, economics, possibly history. Beginnings of a theory of cultural systems.
Symbolic systems	Language, logic, mathematics, sciences, arts, morals, etc.	Algorithms of symbols (e.g. mathe- matics, grammar); 'rules of the game' such as in visual arts and music

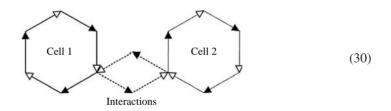
This survey is impressionistic and intuitive with no claim for logical rigour. Higher levels as a rule presuppose lower ones (e.g. life phenomena those at the physico-chemical level, sociocultural phenomena the level of human activity, etc.); but the relation of levels requires clarification in each case (cf. problems such as open system and genetic code as apparent prerequisites of 'life'; relation of 'conceptual' to 'real' systems, etc.). In this sense, the survey suggests both the limits of reductionism and the gaps in actual knowledge. According to the distinction between systems with horizontal cycles and systems with hierarchical cycles, the first three classes of the table are prevalently characterized by horizontal cycles and all the others by hierarchical cycles. This already shows that hierarchical cycles are more spread than one may naively believe. Some entries in the table appear to be imperfectly described, however – for instance, 'biological structures' in the first class above. Similarly, we have doubts about the consistency of the open systems class in the sense that flames present a type of complexity different from the complexity of organisms: a flame is indeed characterized by horizontal cycles, whereas organisms need hierarchical cycles. There are problems with the last class too. The entries there are described *as if* horizontal cycles were sufficient for them (algorithms) whereas none of them can be properly modelled within the boundaries of horizontal cycles only.

4. Relational methodology

To illustrate the relational modelling strategy, we shall first present and discuss two preliminary cases and analyse them on one level only. The two cases are the interaction between two cells and the body-brain connection.

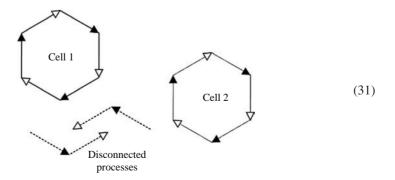
4.1 Interactions between two cells

Let us consider a very simple 'organism' consisting of two cells interacting with each other, thus:



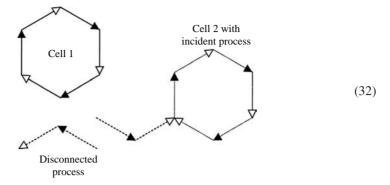
All processes are contained in hierarchical cycles; thus the relational diagram (30) models a clef system L. (In this case, a 'living system'; see discussion in Section 2.7.) Each of cells 1 and 2 is a 'suborganism' of L, since each is a hierarchical cycle.

If we were to fractionate (30), we may obtain



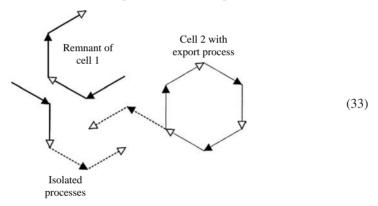
Then, we end up with two living cells, and two disconnected processes.

Alternatively, if the fractionation happens thus:



then, cell 1 is a clef system whereas cell 2, with an out-of-cycle process, but contains a hierarchical cycle, is a complex system. (See definitions in Section 2.7.)

Many other modes of fractionation are possible; for example,

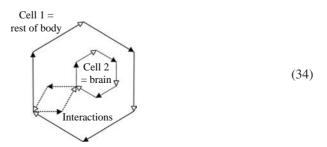


In this case, the fractionation kills cell 1, whereas cell 2 with the export process remains complex.

4.2 The body and the brain

The brain can arguably be considered an organism in its own right, or at least a 'suborganism' of the larger organism to which it belongs. Perhaps the 'interacting organisms' idea from Louie (2010) may be used here, and one can formulate a theory in which the brain (or the central nervous system) and the rest of the body are in a symbiotic relationship.

Now, suppose we consider the living system L in (30) and let cell 2 represent 'brain'. Diagram (30) is topologically equivalent to

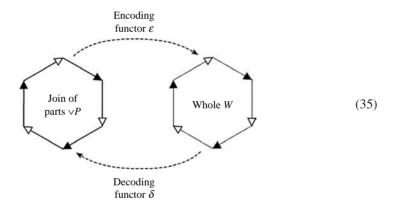


which is a graphic representation of the fact that the brain is a suborganism of L. The 'brain' may be fractionated as an independent organism as in (31). It may be the case that fractionation (32) is a more accurate model, in the sense that an isolated brain is a complex system, but not a living system on its own.

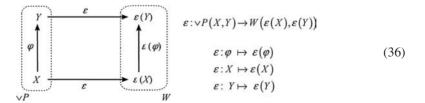
Note that there are no topological difficulties in having 'cycle-within-cycle', and no problems with finding different hierarchical 'subcycles' in an 'overall hierarchical cycle'. What we have drawn are very simple-minded diagrams, of course; but they seem to already capture the essence of the issues at hand. The conclusion is that there are no problems with having a hierarchy of hierarchical cycles (e.g. cells forming a multicellular organism – both levels are living systems). When we fractionate a larger living system into smaller ones, however, some interactions among the smaller ones may be lost.

5. Parts and wholes

Given that impredicative systems – i.e. systems containing hierarchical cycles – do not admit maximal models, we propose to analyse impredicative systems through the *reconciliation of two alternate descriptions* of an impredicative system *S*, a 'join of parts' $\lor P$ (in which at least one part *P* is already an impredicative system) and a 'whole' *W*. Both $\lor P$ and *W* are categories. The two categories $\lor P$ and *W* have different objects and morphisms, but describe the same impredicative system *S* in different modes. The relationship (or interactions) between the two alternate, non-equivalent descriptions is then functors between the two categories. The situation is, of course, the following modelling relation:



In the functorial representation (cf. (8) above), we have



where

 $\lor P$: join of parts; objects = *X*,*Y*; morphisms = $\varphi \in \lor P(X, Y)$ *W*: whole; objects = $\varepsilon(X)$, $\varepsilon(Y)$; morphisms = $\varepsilon(\varphi) \in W(\varepsilon(X), \varepsilon(Y))$ ε : encoding functor

5.1 Examples

For a simple, 'everyday' example, one may consider

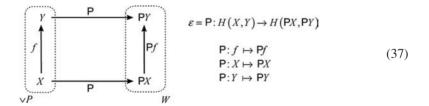
- *S*: system = house
- $\lor P$: objects = materials (bricks, wood, etc) used to build up the house; morphisms = the arrangement of the materials
- *W*: objects = kitchen, dining room, etc.; morphisms = the arrangement of the various rooms
- ε : assemblage of the materials into functional units

Although this exemplification is trivial, it is helpful for illustrative purposes. It serves as a guide for the reader to develop an understanding of the suggested methodology; it also helps in showing that no one-to-one connection between $\lor P$ objects and W objects is implied.

Another way to interpret our $\lor P$ versus W description of systems is in terms of the dichotomy of structure versus function, with $\lor P$ expressing the structural and W expressing the functional subsystems of system S. This dichotomy is evident in the psychological and social system examples below.

Next, let us give a mathematical exemplification. The *power set functor* $P : Set \rightarrow Set$ assigns to each set *X* its power set PX (i.e. the collection of all subsets of *X*), and assigns to each mapping $f : X \rightarrow Y$ the mapping $Pf:PX \rightarrow PY$ that sends each $A \subset X$ to its image $f(A) \subset Y$. One readily verifies that this definition satisfies the functorial requirements $P(g \circ f) = P(g) \circ P(f)$ (the mapping that sends a subset *A* of the domain of *f* to the subset g(f(A)) of the codomain of *g*) and $P1_X = 1_{PX}$ (the identity morphism gets sent to the identity morphism), so P is a covariant functor from **Set** to **Set**.

Now, the entities $\langle X, f \rangle$ and $\langle \mathsf{P}X, \mathsf{P}f \rangle$ are two alternate descriptions of the same system $\langle X \rangle$. The mapping $f : X \to Y$ maps on the 'element level' (i.e. parts): for each $a \in X$ it assigns an image which is an element $f(a) \in Y$. The mapping $\mathsf{P}f : \mathsf{P}X \to \mathsf{P}Y$ maps on the 'set level' (i.e. whole): for each subset $A \subset X$ it assigns an image which is a subset $\mathsf{P}f(A) = f(A) \subset Y$.

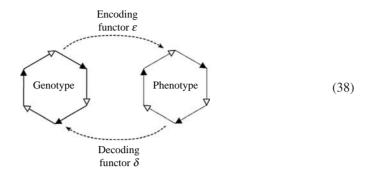


- *S*: system = category **Set**
- ∨*P*: set as a collection of elements: $X = \{a : a \in X\}$; objects = *X*,*Y*; morphisms = $f \in H(X, Y)$
- W: set as lattice of subsets: $X = \sup\{A : A \subset X\} = \bigcup_{A \in \mathsf{P}X} A$; objects = $\mathsf{P}X, \mathsf{P}Y$; morphisms = $\mathsf{P}f \in H(\mathsf{P}X,\mathsf{P}Y)$
- ε : power set functor $\mathsf{P} : \mathbf{Set} \to \mathbf{Set}$

6. Another biologically oriented exemplification

Let us consider another biological example, the phenotype–genotype duality, in some detail. System *S* here is the genetic entity that is an organism, the corresponding $\lor P$ -to-*W*

modelling relation is:



Phenotype is effect, whereas genotype is source. They are dual characteristics that define organisms (any living system in general). The precise formulations of the functors, ε and δ , are of course major problems in genetics, indeed in all of biology. A big question is 'How does genotype determine phenotype?' – the search for the links between the two sides.

Genotype and phenotypes are *non-equivalent* descriptions of a living system. A 'metric' in the space of genotypes is 'closely related'; a 'metric' in the space of phenotypes is 'similar'. D'Arcy Thompson's work may be grossly summarized as 'closely related organisms are similar' – the encoding functor ε then becomes a mapping that 'preserves' the metric structure of the spaces; i.e. a morphism in the (subcategory of metric spaces of the) category **Top** (of topological spaces). A converse question may be posed: 'Are similar organisms closely related?'. In biology, the general answer is no; so the decoding functor δ is not a **Top**-morphism.

Mendel first began the science of genetics as the study of phenotypes. Molecular biologists have, of course, since hijacked the subject into the study of genotypes. This 'substitution' is the very example of reductionism. A specific example is the protein-folding problem, where genotype is the primary sequence, and phenotype is (the active sites on) the native state. Reductionism is especially rampant here when the search is commonly for *algorithmic* realizations of the encoding functor ε . It is little wonder that after decades of efforts, the 'protein-folding algorithm' searcher has not been successful.

The above discussion may be summarized thus: genetics of organism

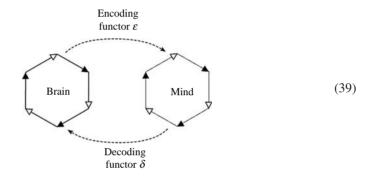
 $\lor P$: genotype; objects = genes; morphisms = 'closely related'

- *W*: phenotype; objects = physiognomy; morphisms = 'similar'
- ε: genetic code

7. The brain-mind connection

The connection between brains and minds does not appear to follow the kind of connections that were, respectively, used for the two-cells and the organism-brain examples (which proved to be topologically analogous). The main problem is that there is a difference between connections among systems of the same nature (such as cells or more generally biological entities) and connections between systems of different nature, such as a biological system (brain) and a psychological system (mind). Minds and their biological bearers are, respectively, 'made' of different materials.

For this reason, the answer appears to be different in kind from the suborganism approach of the brain-body problem. Here is where the alternate description strategy shows its capacity. The standard representation of an organism emphasizes the 'biochemical' (or 'physiological') aspects of metabolism-repair-replication. A 'brain' may conceivably be represented this way, as a 'neuronal organism'. The processes of the 'mind', on the other hand, presumably are not reducible to biochemical or physiological ones. So a hierarchical-cycle representation of the mind would involve completely different maps – let us call them 'psychic maps' for now, for lack of a better term. So, the mind-brain problem may then perhaps be formulated in category theory as the search for functors between the 'neuronal map hierarchical cycle' model of the mind. The following diagram presents the main idea:



'Mind' and 'brain' are each hierarchical cycles in their own right, but with entirely different sets of maps. In 'brain', the maps are more akin to the 'regular' type of biochemical and physiological efficient causation, whereas the nature of the maps in 'mind' will be determined in Section 8.

When different systems are based on widely different types of elements (such as neurons and thoughts), they do not share a common code and, therefore, do not 'understand' each other. The exchanges that occur between them take the form of perturbations. In this sense, different systems *perturb* one another. Each system has its own internal dynamics and generates its own contents. They influence each other not in the form of a direct exchange of information (for the just-mentioned lack of a common code), but as 'disturbances' that the receiving system interprets in its own way. (The same observation can be repeated for other types of structurally coupled systems; one may add that social systems perturb psychological systems, and vice versa, see below.)

According to the two-sided model we are exploiting, the mind-brain problem can then be deciphered as follows:

 $\lor P$: brain; objects = neurons; morphisms = neuronal pathways

- *W*: mind; objects = contents; morphisms = thoughts
- ε : perturbation (in the sense above explained)

8. Psychologically oriented exemplifications

We shall distinguish between 'presentations' and 'representations'. Presentations form what is usually called stream of consciousness, specious present or moment now.

They concern the basic temporal structure of our conscious life. (Immediate) perceptions are the most well-known cases of presentations.

Experimental data show that the following are some of the basic features of presentations (Poli 2006a,b):

- 1. Presentations last from 200 to 3000 μs ca. On average, they last approximately 700 $\mu s.$
- 2. The duration of presentations depends on a variety of factors, ranging from the subject's mood feelings (they are shorter when the subject is excited and longer when she/he is relaxed) to the cognitive state of the subject (attention shortens presentation), to the content of what is presented, etc.
- 3. Presentations come with an inner organization, on various dimensions. Of these the most important are (a) the distinction between focus and periphery, (b) the presence of internal laws of organization, and (c) the elaboration of their content in subsequent stages. Point (a) entails that there are upper limits to the complexity of the correlate in the focus. Point (b) yields possibly most surprising results, namely the laws of temporal and spatial inversion (Benussi 1913). Point (c) claims that presentations themselves have a temporal structure (Albertazzi 2003).
- 4. Presentations come in a (temporal) series, often called stream of consciousness.

Presentations provide the *stuff* (the objects) to be further elaborated by subsequent higher order cognitive acts (e.g. reasoning, imagery, fantasy, and (reactualized) memory). This second level is termed the level of *representations*. These are produced syntheses based on series of presentations. Most recent research on the mind has mainly concerned itself with representations, leaving apart the level of presentation. For this reason, we shall focus here on presentations.

Every psychological act is a threefold entity: it has a source, a target, and a body. The ego is the source of the act, the object is its target, and what connects the ego with the object is the body of the act. The basic structure of an intentional act is thus:



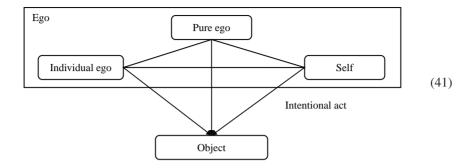
The body and the target of acts are internally linked to one another: for every seeing there is something that is seen, for every thinking there is something that is thought, for every feeling there is something that is felt, etc. The act's objects are internal, not external, objects.

Figure (40) depicts only a minimal part of the real structure of an act. A more satisfactory representation should for instance distinguish the different elements of the ego. At least three components (or substructures) of the ego can accordingly be distinguished: 'pure ego', 'individual ego', and 'self.'

The pure ego is an entirely functional component which in itself does not possess any independent properties. The only feature characterizing the pure ego is that of being the point of origin of intentional acts. In this sense, the pure ego is always present – by definition – in every intentional act. Unlike the pure ego, the individual ego is not exclusively functional. Finally, the self is the public side of the ego, the one that articulates itself in the perception that the subject possesses of the roles that it embodies. The self has mainly to do with forms of socialization (see Poli 2009 for details).

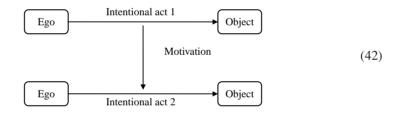
Decomposition of the ego into the three components of pure ego, individual ego and self makes it possible to 'filter' the intentional act in different ways according to the source

or sources concerned, giving a more realistic structure of the intentional act:



In the reality of a completed intentional act, the pure ego, the individual ego, and the self are all involved, and each makes its contribution. The composition of the various arising *triangles* depends in its turn on the composition of the three sides of the ego, that is, of the triangle: Pure ego–individual ego–self.

Motivation is defined as passage from act to act. This provides a graphical representation of the idea:



Motivations unfold according to rules. In this regard, Husserl distinguished between the external and internal horizons of psychological acts. External horizon deals with everything else contemporaneously happening in the field of consciousness. Given a thought, external horizon may move attention

- 1. from the object to its origins,
- 2. from the object to its future potentialities (anticipation),
- 3. from the object to other objects close to it (spatio-temporal continuity), or
- 4. from the object to similar objects (resemblance).

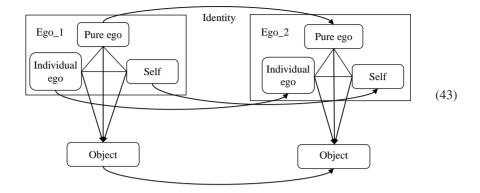
Internal horizon, on the other hand, moves attention from a given object to its structures. Given a thought, internal horizon moves attention

- 1. from the genus to its species (specify) or
- 2. from the species to its genus (generalize).

Although there are other more complex cases, those listed suffices as a first approximation.

Given the decomposition of the ego into the three aspects of pure ego, empirical ego and self, we can redraw diagram (42) to account for the complexity of the ego,





The link between the two pure egos is the link of identity, for obvious reasons. On the other hand, there is no reason to think that links between all the other components of diagram (43) are identity links. This almost geometrical way of analysing the internal structure of intentional acts and motivations as links between acts (and their components) may help shed light on these very complex topics.

Leaving geometric considerations aside, let us return to the crux of our problem. What exactly is meant by saying that motivation is the passage from one act to the following one? The idea is that the ego performs a certain act *because* – on the basis of the fact that – another act has been performed. When I see a thing, I see only one side of it; I may circle around it to see the other sides as well: belief in a certain state of affairs may motivate belief in another state of affairs connected with it; espousing a value may motivate a stance, an act of will, an action. Motivation operates for every type of act, perceptive, evaluative, emotional. I see something beautiful and I feel pleasure at such beauty. All the aspects are involved: perception, recognition, evaluation, and stance taking.

Motivation has a complex structure and we have only scratched its surface. We have seen that both the ego and the object sides of psychological acts play a role in the architecture of motivation. By way of a final summary, the multifaceted articulation of the ego unfolds its own object, which in its turn the object of the act activates the possible ways in which acts follow one another.

The most general structure of the mind can then be captured by the following model:

- $\lor P$: Presentation; objects = (psychological) acts; morphisms = motivation (i.e. the link between a previous act and the subsequent one, following the rules explained above)
- *W*: Representation; objects = subsystems (memory, attention, decision, etc.); morphisms = semantic relevance
- ε: functional closure over presentations (need for completion, either assimilative or additive)

9. Socially oriented exemplifications

We have already said that social systems are systems able to outlive their members. This problem is called the 'reproduction' of a social system. The most obvious answer to the problem of the reproduction of social systems has been provided by Pareto: the reproduction of a social system (its temporal continuity) is brought about by the reproduction of the individuals that happen to make up the system. As obvious as this answer appears,

it nevertheless raises a problem. In fact, it was Parsons who realized that the reproduction of individuals cannot be assumed as a properly sociological category. Although the reproduction of individuals can be seen as a *socially conditioned* problem as one wishes, it nevertheless remains an essentially biological affair. In order to avoid reducing social problems to biological problems, and in order to answer the question of the reproduction of a *social* system satisfactorily, one must find an authentically *social* type of reproduction. Parsons' answer was that the reproduction of a social system is provided by the reproduction of its (social) roles, i.e. by the reproduction of the patterns of action which are typical of that system. The reproduction of a social system is, therefore, the higher order outcome of the reproduction of roles (patterns of action). This answer gives a much firmer basis to social theory. This is not the end of the story, however. Luhmann later came to realize that roles or patterns of action are themselves in need of a firm basis, because roles are implementations of perspective points, interests, values, and – more generally – of meanings. In its turn, the reproduction of roles implies the reproduction of their meanings. In short, the reproduction of a social system is grounded in the reproduction of meaning (Poli 2010).

The second important outcome arising from the series of the three theories we are considering is connected to the question of the basic units of a social system. The question is, Of what is a social system made? Or, What are the elements that make up a social system? The question is much less trivial than it appears. Pareto's answer is the less surprising one: a social system is composed of individual human beings. Agents are the system's units of reproduction. Parsons' answer, instead, is that *roles* are the units of a social system, not agents. Luhmann continues along the path opened by Parsons by adding *meanings* as the units of reproduction of roles.

The proposals of Parsons and Luhmann represent substantial moves towards a dematerialization of social systems. According to both Parsons and Luhmann, social systems are non-material systems; they are *relational* systems *over* a material basis. Neither of them denies that an underlying material basis is needed. The real nature of a social system, however, is not conveyed by its material basis. There is no way to understand what distinguishes social systems from other kinds of systems by studying the biological entities that happen to bear them or the physical environment in which they happen to be embedded. The thesis claims that what is *specifically social* of social systems does not derive from other types of systems, biological or physical. In other words, social systems are higher order systems organized in such a way that their reproduction is governed by the reproduction of properly social units and not by the reproduction of the units that characterize their underlying material bases. The reproduction of a social system requires *authentically social* units of reproduction. Once a material basis has somehow been given, the reproduction of the higher system does follow its own relational laws. Without this theoretical move, sociology cannot be constituted as a science.

Following Luhmann, here is a first trial for contemporary society: social system

- $\lor P$: objects = communications; morphisms = continue the communication
- *W*: objects = functional subsystems; morphisms = perturbations among them
- ε : functional closure over communications

Communications are three-parted acts based on information, utterance, and understanding. Information is the selection of what has to be communicated, utterance is the how of the communication, and understanding refers to what the *receiver* grasps from the previous two aspects of a communication. None of the three components *on its own* is a communication. Only the three components together form a communication, which implies that a communication can never be attributed to any one individual

(Seidl 2005, p. 29). Communications are from the very beginning social acts, for the simple reason that an act of communication requires both a speaker *and* a listener. Communications come in series, one after the other, and form systems of communication. In Luhmann's words: 'As soon as any communication whatsoever takes place among individuals, social systems emerge' (Luhmann 1982, p. 70).

Structurally different types of communications form different structural subsystems within the overall, inclusive social system as a whole. Face-to-face communications form systems of interaction, whereas decisions form organizations. *Functionally* different types of communications form different functional subsystems (economy, policy, law, science, art, etc). Each of them must possess the capacity to distinguish (and, therefore, filter) relevant from irrelevant communications. Apart from this basic capacity, functional subsystems are characterized by specific codes: legal subsystems organize communication along the legal/illegal opposition; political subsystems along the power/non-power opposition; scientific subsystem along the true/untrue opposition, etc. The different codes (usually in the form of a basic opposition) are the source of the functional organization into functional subsystems.

Within a social system, its various functional subsystems become each other's environment. Given that the different functional subsystems are based on different codes, they do not understand each other. As we have already seen, the exchanges that occur between them take the form of perturbations, not the form of an exchange of information.

10. Conclusion

Some of the problems we have touched upon need further development. The most demanding ones are the following:

1. Indications about how to move from the analysis of macrosystems to their subsystems. More examples from biology, cognitive and social sciences should be collected and analysed. Eventually aspects of a general methodology are extracted.

As a start, we collect below a sample of cases exemplifying the generality of the suggested methodology without further comments (the details shall be developed in subsequent reports), other than noting in passing that in each case, the two hierarchical levels of ' ∇P = set as a collection of elements' and 'W = set as lattice of subsets' are evident.

Proteomics

 $\lor P$: 1° sequence; objects = amino acids; morphisms = 'closely related'

- W: (3°) active sites; objects = 2° structures; morphisms = enzymatic action
- ε: protein folding not-an-algorithm

Physiology

- ∨P: cells; objects = cells in a multicellular organism; morphisms = intercellular communication
- *W*: organism; objects = multicellular organisms; morphisms = physiology
- ε: physiologic integration (symbiosis of cells)

Ecology

- ∨P: organisms; objects = organisms in an ecosystem; morphisms = inter-species dynamics
- *W*: ecosystem; objects = ecosystem; morphisms = ecodynamics
- ε: population dynamical equation models

2. A general theory of what were called 'perturbations' is needed. As a starter, two main cases have been distinguished: perturbations between *generically* different systems, such as the brain and the mind, and perturbations between *specifically* different systems such as the political, economical, and legal subsystems of the social system. The latter were called functional subsystems and were constituted along the selection of a specific dual code. It is unclear whether the subsystems of the psychological system (the systems articulating the level of representations, i.e. the reasoning, memory, and planning, subsystems) are purely functional subsystems on a par with the social subsystems. Further analyses are required before arriving at a definite answer.

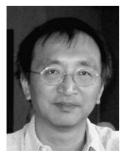
3. Dig deeper into clef systems to see whether they present different layers of higher order complexity. Is the higher order complexity of living systems, psychological systems, and social systems the same? If not, what distinguish the different layers of higher order complexity?

Apart from these still open questions, the main conclusions that arise from this paper can be summarized with the following four theses:

- 1. The vast majority of natural systems includes hierarchical cycles, i.e. they are impredicative or self-referential systems. The generic case is then the case of impredicative systems, not the case of predicative system. Apparently, only a meagre fraction of physical systems are predicative.
- 2. Systems characterized by the presence of hierarchical cycles have properties remarkably different from systems without hierarchical cycles. In short: they do not have largest models, they are not fractionable and they contain at least one non-simulable model.
- 3. The topological network of entailment processes and their hierarchical cycles determines the different types of complexity.
- 4. Understanding hierarchical cycles helps understanding some of the major obstructions of contemporary science, such as the brain-mind conundrum and the problem of the interaction between psychological and social systems. Both represent cases of interactions between systems of a different nature. Hierarchical cycles appear to be one of the very few ideas able to break the divide that isolates and fragment the various sciences.

During the past century, the idea of a structurally non-reductionistic science has been raised time and again, most of the times, however, without any robust grounding in science and mathematics. The research programme arising from the framework of hierarchical cycles appears to be able to properly contribute to such a demanding – and most needed – objective.

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(vol. 39, no. 7, October 2010, pp. 793–796). Louie's premier interest is in pure mathematical biology: conception and abstract formulations. His two major research topics are category-theoretic aspects of living systems and a phenomenological calculus based on multilinear algebra. He was glad to return to them in 2005 after a twenty-year interlude of doing routine things in mathematical modelling and computer simulations of various physical phenomena, as a scientist-for-hire. Louie also dabbles in the arts, as a choral tenor and a graphic designer.



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References

- Albertazzi, L., 2003. From Kanizsa back to Benussi: varieties of intentional reference. *Axiomathes*, 13 (3–4), 239–259.
- Baianu, I.C., Glazebrook, R. and Brown, J.F., 2011. A category theory and higher dimensional algebra approach to complex systems biology, meta-systems and ontological theory of levels: emergence of life, society, human consciousness and artificial intelligence. *In*: Barna Iantovics, *et al.*, eds. *Acta Universitatis Apulensis, Alba Iulia: conference proceedings: understanding intelligent and complex systems.*

Benussi, V., 1913. Psychologie der zeitauffassung. Heidelberg: Winter.

Boulding, K.E., 1956. The image. Ann Arbor, MI: University of Michigan Press.

- Ehresmann, A.C. and Vanbremeersch, J.-P., 2007. *Memory evolutive systems: hierarchy, emergence, cognition*. Amsterdam: Elsevier.
- Louie, A.H., 2009. More than life itself: a synthetic continuation in relational biology. Frankfurt: Ontos Verlag.
- Louie, A.H., 2010. Relational biology of symbiosis. Axiomathes, 20 (4), 495-509.
- Luhmann, N., 1982. The differentiation of society. New York: Columbia University Press.
- Poli, R., 2001. The basic problem of the theory of levels of reality. Axiomathes, 12 (3-4), 261-283.
- Poli, R., 2006a. Levels of reality and the psychological stratum. *Revue internationale de philosophie*, 61 (2), 163–180.
- Poli, R., 2006b. First steps in experimental phenomenology. In: A. Loula, R. Gudwin and J. Queiroz, eds. Artificial cognition systems. Hersey, PA: Idea Group Publishing, 358–386.
- Poli, R., 2007. Three obstructions: forms of causation, chronotopoids, and levels of reality. *Axiomathes*, 17 (1), 1–18.
- Poli, R., 2009. Causality and motivation. *In*: R. Poli, ed. *Causality and motivation*. Frankfurt: Ontos Verlag, 7–22.

- Poli, R., 2010. The complexity of self-reference. A critical evaluation of Luhmann's theory of social systems. *Journal of sociocybernetics*, in press.
- Seidl, D., 2005. The basic concepts of Luhmann's theory of social systems. *In*: D. Seidl and K.H. Becker, eds. *Niklas Luhmann and organization studies*. Copenhagen: Liber & Copenhagen Business School Press, 21–53.

von Bertalanffy, L., 1968. General system theory. New York: George Braziller, Inc.