Mathematical Foundations of Anticipatory Systems

A. H. Louie

Abstract

A natural system is an anticipatory system if it contains an internal predictive model of itself and its environment, and in accordance with the model's predictions, antecedent actions are taken. An organism is the very example of an anticipatory system. Deep system-theoretic homologies allow the possibility of obtaining insights into anticipatory processes in the human and social sciences from the understanding of biological anticipation. To this end, a comprehensive theory of anticipatory systems is the means. The present chapter is an exposition on the mathematical foundations of such a theory.

Keywords

Robert Rosen • Relational biology • Anticipatory system • Modelling relation • Encoding • Decoding • Causality • Inference • Commutativity • Category theory • Functor • Simulation • Model • Analogue • Conjugacy • Surrogacy • Internal predictive model • Antecedent actions • Transducer • Feedforth

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Historia

Robert Rosen formulated his theory of anticipatory systems during the academic year 1971–72, when he was a visiting fellow at the now-defunct Center for the Study of Democratic Institutions (CSDI) in Santa Barbara, California. The term 'anticipatory system' first appeared in his publications in the paper "Planning, Management, Policies and Strategies: Four Fuzzy Concepts" (Rosen 1974, which was first incarnated as an internally circulated CSDI Discussion Paper scheduled for the Center's 'Dialog' on Tuesday, 16 May 1972). He defined therein the "anticipatory modes of behavior of organisms" to be those

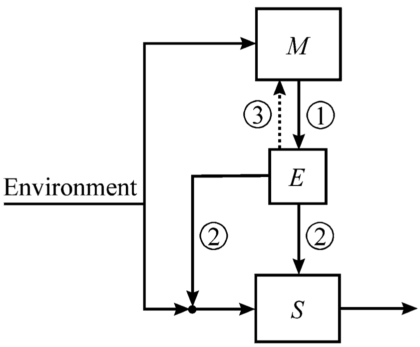
> in which an organism's present behavior is determined by (a) sensory information about the present state of the environment, and (b) an 'internal model' of the world, which makes predictions about future states on the basis of the present data and the organism's possible reactions to it.

It was also in this paper that the now-iconic diagram of an anticipatory system first appeared in Rosen's writings (Fig. 1).

In Fig. 1, *S*, *M*, and *E* are, respectively, object system, predictive model, and set of effectors. (I shall have more to say about this diagrammatic representation later on in this chapter.)

Biology is abounding with situations in which organisms can generate and maintain internal predictive models of themselves and their environments, and use the predictions of these models about the future for purpose of control in the present. This is true at every level, from the molecular to the cellular to the physiological to the behavioral, and this is true in all parts of the biosphere, from microbes to plants to animals to ecosystems. One may succinctly postulate the

Fig. 1 Anticipatory system 1.1.1



Axiom of Anticipation Life is anticipatory.

But anticipatory behavior, while a necessary condition for life, is not restricted to the biological universe; it, indeed, encompasses the "four fuzzy concepts" and more. At the human level it can be multiplied without end and may seem fairly trivial: examples range from avoiding dangerous encounters to strategies in games and sports, including those of economics and politics.

Although the *concept* of 'anticipation' had not been new, the *systemic study* of it was when Rosen wrote his 1974 paper. Rosen's rigorously mathematical study of this biology-inspired subject led to a sequence of papers, culminating in his book *Anticipatory Systems: Philosophical, Mathematical, and Methodological Foundations* (Rosen 1985a; henceforth denoted by the symbol *AS*). Therein, in Section 6.1, is the generalized, formal

Definition An *anticipatory system* is a natural system that contains an internal predictive model of itself and of its environment, which allows it to change state at an instant in accord with the model's predictions pertaining to a later instant.

An anticipatory system's present behavior depends upon 'future states' or 'future inputs' generated by an internal predictive model. 'Model-based behavior' (or more specifically 'anticipatory-model-based behavior') is the essence of social, economic, and political activity. Beyond its organismic origins, an anticipatory system finds analogies in social systems, economics, politics, ethics, future studies, and many others. The common question in these diverse fields is that of *ought*, which may be phrased as "What should we do now?". However different the contexts in which the question is posed, *they are all alike in their fundamental concern with the making of policy, and the associated notions of forecasting the future and planning for it*. What is sought, in each of these diverse areas, is in effect a strategy of decision making. An understanding of the characteristics of model-based behavior is thus central to any strategy one wishes to develop to control and manage such systems, or to modify their model-based behavior in new ways. But underlying any strategy, there must be an underlying substratum of basic principles: a science, a *theory*. Rosen proposed that the theory underlying a strategy of policy generation is that of anticipatory systems.

Note, in contrast, that a *reactive system* can only react, in the present, to changes that have already occurred in the causal chain, while an *anticipatory system*'s present behavior involve aspects of past, present, and future. The presence of a predictive model serves precisely to pull the future into the present; a system with a 'good' model thus behaves in many ways as if it can anticipate the future. In other words, *a predictive model permits anticipation*. Indeed, to use teleological language, the *purpose* of a predictive model is to anticipate. The 'anticipatory paradigm' *extends – but does not replace* – the 'reactive paradigm', which has dominated the study of natural systems, and allows us a glimpse of new and important aspects of system behavior.

Robert Rosen was a mathematical biologist. Anticipation is a *necessary* condition of life: a living system anticipates. This is the connection that explains how Rosen, in his lifelong quest of general principles that would answer the question "What is

Life?", happened to write, en passant, 'the book' *AS* on anticipation. The chapter "Relational Biology" in this *Handbook* explains the placement of anticipation in the context of this quest. For an expository introduction to Robert Rosen's anticipatory systems, the enthused reader may like to consult Louie (2010).

For emphasis, let me restate the definition of anticipatory system thus:

Definition An *anticipatory system* is a natural system that contains an

$$\textit{internal} \bullet \textit{predictive} \bullet \textit{model} \tag{1}$$

of itself and of its environment, which allows it to

$$\textit{change} \text{ state at an instant } \textit{in accord with the model's predictions} \tag{2}$$

pertaining to a later instant.

Both (1) *and* (2) are crucial ingredients of anticipation. Having a model of the future is not enough: although the entailment *process* of anticipation is embedded in an anticipatory system's defining component *object* (1), an anticipatory system must also (2) make use of the prediction and take proactive antecedent actions accordingly. True to the spirit of *relational biology* (*cf.* the chapter in this *Handbook* so named), the crux in this definition is not what an anticipatory system itself *is*, but what it *does*.

The Modelling Relation

A *model* is the representation of one system in another. The only perfect model of a system is, however, the system itself. Otherwise, by genericity, by necessity, by practicality, and indeed by definition, a model is *incomplete* and only captures a proper subsystem.

Category theory is, among its many roles, the general mathematical theory of *modelling* and, incidentally, the metalanguage of relational biology. The Appendix of Louie (2009) is a terse summary. The definitive reference on this branch of abstract algebra remains the quintessential Mac Lane (1997), to which I refer the reader for further exploration of the category-theoretic topics that I am presenting in this chapter. Category theory has metamorphized from its origins in the early 1940s, when its founders Samuel Eilenberg and Saunders Mac Lane applied it to algebraic topology (which may be considered as modelling geometry in algebra). It is now an autonomous research area in contemporary mathematics and has metastasized into theoretical computer science, mathematical physics, and, yes, mathematical biology. Relational biology is, however, not the only approach in mathematical biology that uses category theory; other notable approaches are, to name but two, that of the late Ion Baianu (*cf.* Brown and Glazebrook 2013) and Memory Evolutive Systems of Andrée Ehresmann and Jean-Paul Vanbremeersch (*cf.* the chapter "▶ Anticipation in MES-Memory Evolutive Systems" in this *Handbook*).

The category in which the collection of objects is the collection of all sets (in a suitably naive universe of small sets) and where morphisms are (single-valued) mappings is denoted **Set**. The category in which the collection of objects is the collection of all sets (in a suitably naive universe of small sets) and where morphisms are set-valued mappings (equivalently, relations) is denoted **Rel**. (The first five chapters of Louie 2013 is an introduction to the theory of set-valued mappings, and the rest of the book is on their implications in biology.) For sets X and Y, the hom-sets **Set**(X, Y) and **Rel**(X, Y) contain, respectively, *all* single-valued mappings and *all* set-valued mappings from X to Y. **Set**(X, Y) is a proper subset of **Rel**(X, Y); **Set** is a non-full subcategory of **Rel**.

Modelling is the art that is the ultimate revelatory reflection of life. Having an internal model of the world is a characterization of living systems; sentience is not a prerequisite for this necessary fundamental property. With the animation into sentience and the evolution into consciousness, however, modelling gains a volitional dimension. The teleological representation of one system in another is an ancient human enterprise: one finds patterns and ever theorizes. Even the meta-modelling of modelling itself, i.e., to methodologically study modelling *qua* modelling as opposed to just make models, can trace its genealogy back to antiquity. The epistemology of modelling involved scholars from Plato and Aristotle, through Kepler and Galileo, to Newton, von Helmholtz, Mach, Hertz, Bohr, and many others.

Heinrich Hertz, in the introduction of his posthumously published masterwork *Die Prinzipien der Mechanik in neuem Zusammenhange dargestellt* (1894; English translation *The Principles of Mechanics Presented in a New Form*, Hertz 1899), gave the following meta-model:

> We form for ourselves images or symbols of external objects; and the form which we give them is such that the necessary consequents of the images in thought are always the images of the necessary consequents in nature of the things pictured. In order that this requirement may be satisfied, there must be a certain conformity between nature and our thought. Experience teaches us that the requirement can be satisfied, and hence that such a conformity does in fact exist. When from our accumulated previous experience we have once succeeded in deducing images of the desired nature, we can then in a short time develop by means of them, as by means of models, the consequences which in the external world only arise in a comparatively long time, or as the result of our own interposition.

A model is an image in thought of an external object, with a certain conformity between nature and thought: this is the essential meta-model Hertz communicated in his introductory paragraphs before launching into the *raison d'être* of his book, the Archimedes–Galileo–Newton–Lagrange–d'Alembert models of mechanics.

The endeavor of meta-modelling culminates in Robert Rosen's modelling relation, a functorial (in the category-theoretic sense) diagrammatic representation he first introduced in 1979 when he wrote *AS*. Modelling and meta-modelling are essential ingredients in *AS* – to wit, chapter 3 is entitled "▶ The Modelling Relation" and chapter 5 is entitled "▶ Open Systems and the Modelling Relation". Indeed, the main theoretical questions with which one deals in *AS* are (as Rosen summarized in the Foreword of *AS*):

(a) What is a model?
(b) What is a *predictive* model?
(c) How does a system which contains a predictive model differ in its behavior from one which does not?

A *modelling relation* is a commutative functorial encoding and decoding between two systems. Between a natural system (an object partitioned from the physical universe) N and a formal system (an object in the universe of mathematics) M, the situation may be represented in the following diagram (Fig. 2).

Causal entailment is the manifestation of the Aristotelian efficient cause in natural systems, and correspondingly inferential entailment is the manifestation of the Aristotelian efficient cause in formal systems. The εncoding ε maps the natural system N and its causal entailment c therein to the formal system M and its internal inferential entailment i; i.e.,

$$\varepsilon : N \mapsto M \quad \text{and} \quad \varepsilon : c \mapsto i. \tag{3}$$

The δecoding δ does the reverse. (The (ε, δ) notation in Fig. 2 is my allusion to the (ε, δ) argument of mathematical analysis; viz. "$\forall \varepsilon > 0 \quad \exists \delta > 0 \cdots$.") The entailments satisfy the commutativity condition that tracing through arrow c is the same as tracing through the three arrows ε, i, and δ in succession. This may be symbolically represented by the 'composition'

$$c = \delta \circ i \circ \varepsilon. \tag{4}$$

Stated otherwise, one gets the same answer whether one, as in the left-hand side of (4), simply sits as observers and watch the unfolding sequence of events c in the natural system, or, as in the right-hand-side of (4), (i) encodes ε some properties of the natural system into the formalism, (ii) uses the implicative structure i of the formal system to derive theorems, and then (iii) decodes δ these theorems into propositions (*predictions*) about the natural system itself. When the commutativity (4) holds, one has established a congruence between (some of) the causal features of the natural system and the implicative structure of the formal system. Thence related, M is a *model* of N, and N is a *realization* of M.

One may possibly construct parts of Fig. 2 from the brief Hertz passage on "images" in his introduction to *The Principles of Mechanics*, but this is not sufficient

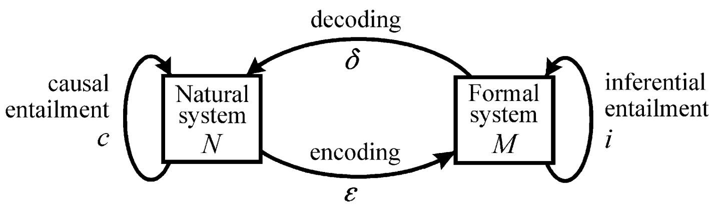


Fig. 2 The prototypical modelling relation

to give Hertz precedence over Rosen in the formulation of the modelling relation. While Hertz had the correct principles of meta-modelling (so did a host of scholars before him), he did not produce any diagrams. Indeed, although *The Principles of Mechanics* contained an abundance of mathematical formulae and equations, it had no diagrams whatsoever. Rosen's category-theoretic rendering of the modelling relation as a commutative arrow diagram is his unique contribution. In any case, when Hertz wrote his book, the birth of category theory (1945) was 50 years in the future. So when one engages the modelling relation in terms of the functorial diagram that is Fig. 2, one is clearly dealing with *Robert Rosen's* modelling relation.

I emphasize that the attribution of the modelling relation to Rosen was not only because he drew (a predecessor of) the arrow diagram Fig. 2. Block diagrams that connect boxes with a flow of arrows had long been in existence, and they may be considered subject-oriented specializations of directed graphs. Examples of these block diagrams include component interaction diagrams in control theory, flow diagrams in computer programming, and schematic diagrams in engineering, and they all contain simple specimens isomorphic in form to Fig. 2. Rosen's originality was in presenting the levels of entailment (3) and the compositional commutativity (4) in his meta-model from a category-theoretic standpoint.

Let me indulge in a bit of historic trivia on the evolution of the arrow diagram Fig. 2. Rosen explicated the modelling relation in detail in his 1979 draft of *Anticipatory Systems* (*AS*: 2.3), and the first arrow diagram was Fig. 2.3.1 therein (Fig. 3).

Note that in this inaugural version, causality in the left-hand-side natural system was not yet part of the formulation. A variety of mundane nuisances delayed the publication of *AS* until 1985; meanwhile, Rosen marched on with his meta-modelling. By the first time the arrow diagram of the modelling relation appeared in print, as Fig. 1 on *p*.91 of Rosen (1980), the dual processes of "system behaviors" (causality) and "rules of reference" (inference, implication) were in place (Fig. 4).

The modelling relation diagram made another pre-*AS*-publication appearance in Rosen (1985b: Fig. 1 on *p*.179) (Fig. 5).

By this juncture in 1985, all three key ingredients of the modelling relation were present: (i) the correspondence of objects

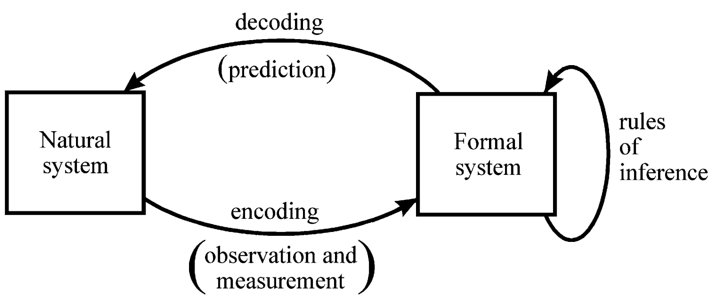


Fig. 3 Modelling relation, version 1 (*AS* Figure 2.3.1)

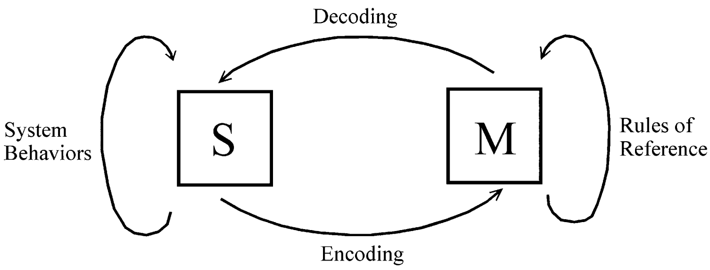


Fig. 4 Modelling relation, version 2

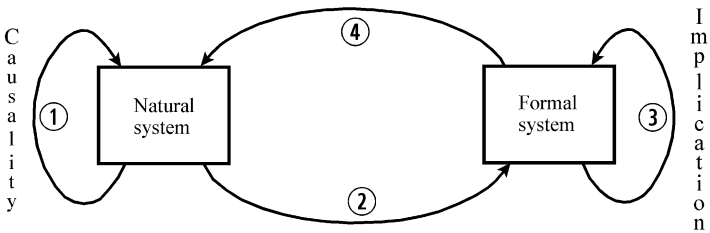


Fig. 5 Modelling relation, version 3

$$\text{Natural system} \underset{\text{Encoding}}{\overset{\text{Decoding}}{\leftrightarrows}} \text{Formal system} \tag{5}$$

(ii) the correspondence of morphisms

$$\text{Causality} \underset{\text{Encoding}}{\overset{\text{Decoding}}{\leftrightarrows}} \text{Implication} \tag{6}$$

and (iii) the commutativity

$$\text{arrow } \textcircled{1} = \text{arrows } \textcircled{2} + \textcircled{3} + \textcircled{4} \tag{7}$$

(*cf.* (3) and (4) above). (Note, however, that there was no retro-editing of *AS*'s diagrams: all modelling-relation diagrams in *AS* were published without the left-hand-side causality arrows.)

Figure 5 is the form of the functorial representation of the modelling relation that Rosen would use henceforth (and almost always with the idiosyncratic – and typesetting unfriendly – 'circled number' labels for the arrows). He revisited epistemological considerations of meta-modelling in *Life Itself* (Rosen 1991; notably Section 3H on "The Modeling Relation and Natural Law") and in *Essays on Life Itself* (Rosen 2000; in particular chapter 10 "▶ Syntactics and Semantics in

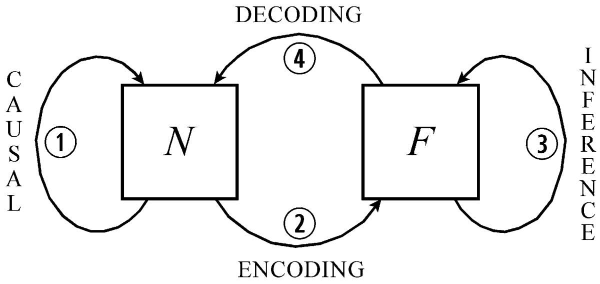


Fig. 6 Modelling relation, version 4

Languages"); a variant of the arrow diagram appeared therein, respectively, as Fig. 3H.2 and Fig. 10.1 (Fig. 6).

Natural Law

Natural order is woven into the fabric of reality. *Causality* is the principle that every effect has a cause and is a reflection of the belief that successions of events in the world are governed by definite relations. *Natural Law* posits the existence of these *entailment* relations *and* that this causal order can be *imaged* by implicative order. *System* is a basic undefined term, a primitive. It takes on the intuitive meaning of 'a collection of material or immaterial things that comprises one's object of study'. In relational, hence nonmaterial, terms, a system may be considered as a *network of interacting processes*.

In terms of the modelling relation (Fig. 2), Natural Law is an *existential declaration* of causal entailment c and the encodings $\varepsilon : N \mapsto M$ and $\varepsilon : c \mapsto i$ (*cf.* (3) above). A formal system may simply be considered as a *set* with additional mathematical structures. So the mathematical statement $\varepsilon : N \mapsto M$, i.e., the posited existence for every natural system N a model formal system M, may be stated as the axiom

$$\textit{Everything is a set.} \tag{8}$$

Causal entailment in a natural system is a network of mutually entailing efficient causes. The mathematical statement $\varepsilon : c \mapsto i$, i.e., the functorial correspondence of morphisms, between causality c in the natural domain and inference i in the formal domain, may thus be stated as an epistemological principle, the axiom

$$\textit{Every process is a (set-valued) mapping.} \tag{9}$$

[See the exposition in Louie (2015) for the extension of (9) from "Every process is a mapping." (Louie 2009) to "Every process is a set-valued mapping."] Together, the two axioms (8) and (9), formalizing, respectively, the material and functional parts of nature, are the mathematical foundation of Natural Law. They are manifest in Wigner's observation of "the unreasonable effectiveness of mathematics in the natural sciences". This wedding of mathematics to empiricism has the status of a

Euclidean 'Common Notion', a general logical principle that proclaims the mathematical nature of the world and its fundamental correspondence to our cognitive modes.

Axioms (8) and (9) serve to alternatively characterize a system as a network of interacting (set-valued) mappings and therefore put our operation theater of modelling in the category **Rel** of sets and set-valued mappings. (I must mention in passing that category theory has many flavors; some of which are not set-based. Axioms (8) and (9) are the axioms of 'our flavor' of category theory for relational biology.) In our practice, then, models are drawn from smaller non-full subcategories **C** of **Rel**, in which **C**-objects are a selection of sets A, B, . . ., and **C**-hom-sets $\mathbf{C}(A, B)$ are proper subsets of $\mathbf{Rel}(A, B)$:

$$\mathbf{C}(A,B) \subset \mathbf{Rel}(A,B) = \mathcal{P}(A \times B). \tag{10}$$

The collection of all models of a system N is denoted $\mathbf{C}(N)$. $\mathbf{C}(N)$ is a lattice as well as a category. The category $\mathbf{C}(N)$ is a subcategory of **C**, the source of our modelling sets and mappings (Louie 2015). Let $\kappa(N)$ be the collection of all efficient causes in N. An entailment network that models N may be denoted $\varepsilon(N) \in \mathbf{C}(N)$; the morphism correspondence $\varepsilon : \kappa(N) \to \kappa(\varepsilon(N))$ implies $\varepsilon(\kappa(N)) \subset \kappa(\varepsilon(N))$. Natural Law is the predicate calculus statement

$$\begin{aligned} &\forall N \;\; \exists\, \varepsilon \;\; \exists\, M \in \mathbf{C}(N) \;\; : \;\; M = \varepsilon(N) \\ &\qquad \wedge \;\; \forall\, c \in \kappa(N) \;\; \exists\, i \in \kappa(M) \;\; : \;\; i = \varepsilon(c). \end{aligned} \tag{11}$$

Let $\mathcal{O}\mathbf{C}$ be the collection of **C**-objects (that are sets) and $\mathcal{A}\mathbf{C}$ be its collection of **C**-morphisms (that are set-valued mappings). Thus a **C**-object is $A \in \mathcal{O}\mathbf{C}$ (although a slight notational imprecision may permit $A \in \mathbf{C}$) and a **C**-morphism F belonging to a **C**-hom-set $\mathbf{C}(A, B)$ is $F \in \mathbf{C}(A, B) \subset \mathcal{A}\mathbf{C}$. A model of N in the category **C** may be described as a formal system that is a network of mappings in $\mathcal{A}\mathbf{C}$, whence

$$\varepsilon(N) \subset \mathcal{O}\mathbf{C} \quad \text{and} \quad \varepsilon(\kappa(N)) \subset \mathcal{A}\mathbf{C}. \tag{12}$$

For notational simplicity, one often drops the encoding symbol ε and uses N to denote both the natural system and its network model that is a formal system. Thus 'an entailment network $\varepsilon(N)$ that models a natural system N' abbreviates to 'an entailment network N'. Likewise the symbol $\kappa(N)$ shall denote the collection of efficient causes in both the natural system and the formal system. These identifications

$$\varepsilon(N) = N \quad \text{and} \quad \varepsilon(\kappa(N)) = \kappa(N) \tag{13}$$

amount to an implicit invocation of Natural Law, substituting systems with their functorial images. I shall presently explore the consequences of this correspondence.

The canonical modelling relation (Fig. 2) provides a concrete embodiment of Natural Law. But the relation may be generalized, so that the systems N and M may both be natural systems or both be formal systems, and the entailments c and i are corresponding efficient causes; i.e., the modelling relation may simply be a

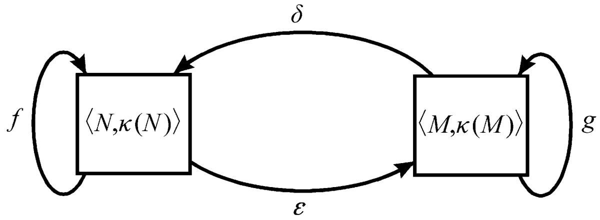


Fig. 7 General modelling relation

commutative diagram between 'general systems'. The general modelling relation has multifarious manifestations, e.g., category theory, analogies, alternate descriptions, similes, metaphors, and complementarities (Louie 2009, sections 4.16–4.20).

A system N, whether natural or formal, considered as a network of interacting processes, may be represented by the pair $\langle N, \kappa(N)\rangle$. Note that in the previous sentence, the symbol N is used for both the system and its underlying collection of sets, whence 'a system N' instead of 'a system $\langle N, \kappa(N)\rangle$'. This convention in mathematical usage will normally give rise to no confusion; one uses the latter pair representation when the context requires it for clarity. In the context of the equivalence imputed by this convention and the abbreviation (13), one has

$$N \subset \mathcal{O}\mathbf{C} \quad \text{and} \quad \kappa(N) \subset \mathcal{A}\mathbf{C}. \tag{14}$$

The general modelling relation in its minimalist category-theoretic form is the arrow diagram thus (Fig. 7).

The Many Levels of the Encoding Functor

True to its category-theoretic taxonomy as *functor*, the encoding ε maps on many levels. On the category-of-models level,

$$\varepsilon : N \to \mathbf{C}(N). \tag{15}$$

The encoding functor ε assigns to each representation $\langle N, \kappa(N)\rangle$ of N a model system

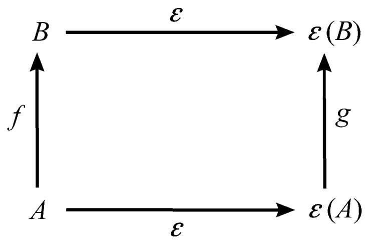
$$\langle M, \kappa(M)\rangle = \langle \varepsilon(N), \varepsilon(\kappa(N))\rangle \tag{16}$$

in $\mathbf{C}(N)$. The assignment $\varepsilon : N \mapsto M$ is a *choice mapping* that singly selects, as a specific model of the natural system N, the formal system M from the set $\mathbf{C}(N)$. But in addition to this *set-pairing* $(N, M) \in \varepsilon$, ε also functions on the *point-pairing* level as a mapping

$$\varepsilon : N \mapsto M \tag{17}$$

from one set into another – to each input element (material cause) $n \in N$, there corresponds a unique output element (final cause) $m \in M$ such that $(n, m) \in \varepsilon$; i.e., $\varepsilon : n \mapsto m$.

Let $f : A \to B$ be a mapping representing a process in the entailment structure of $\langle N, \kappa(N)\rangle$. Suppose there is a mapping $g : \varepsilon(A) \to \varepsilon(B)$ (which is a process in the entailment structure of $\langle M, \kappa(M)\rangle$) that makes the diagram



(18)

commute; i.e., the equality of sequential compositions

$$\varepsilon \circ f = g \circ \varepsilon, \tag{19}$$

or, what is the same,

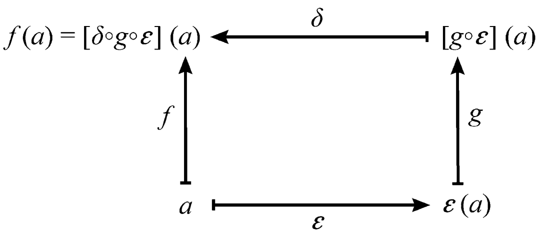
$$\varepsilon(f(a)) = g(\varepsilon(a)) \tag{20}$$

holds for all $a \in A$. Note that this commutativity condition places no further restrictions on the mapping g itself, other than that the two compositions need to reach a common final destination. Such emphasis on the results *regardless of the manner in which they are generated* (i.e., with no particular concern on underlying principles) is the case when $\langle M, \kappa(M)\rangle$ is a *simulation* of $\langle N, \kappa(N)\rangle$.

Together with the decoding δ, the commutativity condition (4) when applied to the general modelling relation of Fig. 7 is

$$f = \delta \circ g \circ \varepsilon\,, \tag{21}$$

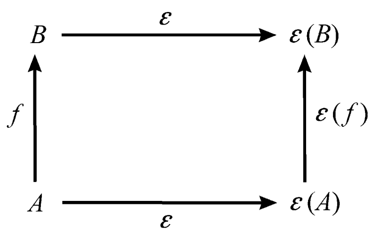
and may be drawn as the element trace



(22)

This commutativity condition involves all four arrows in Fig. 7 and may be stated as 'whether one follows *path f* or *paths* ε , *g* , δ *in sequence*, one reaches the same destination', but, again, it places no further restrictions on the mapping g itself. Systems $\langle N, \kappa(N)\rangle$ and $\langle M, \kappa(M)\rangle$ thus related are called *surrogate*s of each other (and that they satisfy a *surrogacy relation*).

If, in addition, the mapping g is *itself* entailed by the encoding ε, i.e., if $g = \varepsilon(f)$, whence the mapping in $\langle M, \kappa(M)\rangle$ is $\varepsilon(f) : \varepsilon(A) \to \varepsilon(B)$, then instead of (18), one has the commutative diagram



(23)

and the equality corresponding to (20), for every element $a \in A$, is

$$\varepsilon(f(a)) = \varepsilon(f)(\varepsilon(a)). \tag{24}$$

When this more stringent condition (24) is satisfied, the simulation is called a *model*. If this *modelling relation* is satisfied between the systems $\langle N, \kappa(N)\rangle$ and $\langle M, \kappa(M)\rangle$, one then says that there is a *congruence* between their entailment structures, and that $\langle M, \kappa(M)\rangle$ is a *model* of $\langle N, \kappa(N)\rangle$. The element-trace $\varepsilon : f \mapsto g$ is a functorial correspondence of morphisms

$$\varepsilon : \kappa(N) \to \kappa(M). \tag{25}$$

This *process-pairing* $(f, g) \in \varepsilon$ functions on a higher hierarchical level than point-pairing, because now the output is *itself* a mapping $g = \varepsilon(f) \in \kappa(M)$. Together with the decoding δ, the commutativity condition (21) for a model is

$$f = \delta \circ \varepsilon(f) \circ \varepsilon. \tag{26}$$

A simulation of a process provides a description of the entailed effects, whereas a model requires more: a model is a special kind of simulation that additionally also provides a description of the entailment structure of the mapping representing the process itself. A simulation describes only one set of effects, but in itself reveals little about the operative forces, and therefore tells nothing about what would have happened under slightly different circumstances (in which case an entirely different simulation might very well be required). A model, on the other hand, entailing both structure and function of the effects, is structurally stable, and hence 'nearby' (in an appropriate topological sense) effects would have similar models (with perhaps slightly different constitutive parameters).

The encoding functor ε of a model thus encompasses both kinds of entailment in its effects: the output $b = \varepsilon(a) \in M$ is an object, and $\vdash b$ is *material entailment*; the output $g = \varepsilon(f) \in \kappa(M)$ is a mapping, and $\vdash g$ is *functional entailment*. (See the chapter "▶ Relational Biology" in this *Handbook* for explications of material entailment, functional entailment, and the entailment symbol $\vdash$.)

The category **S** of formal systems is the subject of Chapter 7 of Louie 2009. An **S**-object is a pair $\langle X, \mathrm{K} \rangle$, where X is a set and $\mathrm{K} \subset \cdot^{X}$ is a collection of mappings with domain X (*cf.* axioms (8) and (9)). A general system $\langle N, \kappa(N) \rangle$ is a finite collection $\{\langle X_i, \mathrm{K}_i \rangle : i = 1, \ldots, n\}$ of interconnected **S**-object components. N is the collection of sets $N = \{X_i : i = 1, \ldots, n\}$, whence, for each $i \in \{1, \ldots, n\}, X_i \in N$. The relational diagram of $\kappa(N)$ is a digraph representing the entailment network N; for each $i \in \{1, \ldots, n\}$, $\mathrm{K}_i \subset \kappa(N)$, but $\kappa(N)$ may also contain inter-component mappings, e.g., $F : X_i \to X_j$ with $i \neq j$.

The many operational levels of the encoding functor ε (and, by converse induction, of the decoding functor δ) are succinctly manifested in their roles as system morphisms:

$$\langle N, \kappa(N) \rangle \underset{\varepsilon}{\overset{\delta}{\leftrightarrows}} \langle M, \kappa(M) \rangle \tag{27}$$

Recall that a model is almost always incomplete, so it generally cannot be a model of the whole system $\langle N, \kappa(N) \rangle$, but, rather, a proper subsystem $\langle H, \kappa(H) \rangle \subset \langle N, \kappa(N) \rangle$. In view of the Natural Law statement (11) and containments (14), encoding ε entails, for each set $A \in H \subset N \subset \mathcal{O}\mathbf{C}$ and for each mapping $F \in \mathbf{C}(A, B) \subset \kappa(H) \subset \kappa(N) \subset \mathcal{A}\mathbf{C}$, that

$$\begin{aligned} &\varepsilon : A \mapsto \varepsilon(A) \in \varepsilon(H) = M \subset \mathcal{O}\mathbf{C} \\ &\varepsilon : F \mapsto \varepsilon(F) \in \mathbf{C}(\varepsilon(A), \varepsilon(B)) \subset \kappa(M) \subset \mathcal{A}\mathbf{C}\,. \end{aligned} \tag{28}$$

This is the sense of the functorial encoding of $\langle N, \kappa(N) \rangle$ into its model $\langle M, \kappa(M) \rangle$: the encoding functor $\varepsilon : \langle N, \kappa(N) \rangle \to \langle M, \kappa(M) \rangle$ is operationally the restriction $\varepsilon|_{\langle H, \kappa(H) \rangle} : \langle H, \kappa(H) \rangle \to \langle M, \kappa(M) \rangle$. With this understanding of 'incomplete models', however, for simplicity of notation and metalanguage, one simply drops the reference to the subsystem. Incidentally, the subsystem $\langle H, \kappa(H) \rangle \subset \langle N, \kappa(N) \rangle$ is a model of the system $\langle N, \kappa(N) \rangle$; the encoding is the (restriction of) the identity functor $\iota|_{\langle H, \kappa(H) \rangle} : \langle H, \kappa(H) \rangle \to \langle H, \kappa(H) \rangle$. And, as I mentioned at the outset, the only *perfect* model of a system is the system itself, the trivial encoding being $\iota : \langle N, \kappa(N) \rangle \to \langle N, \kappa(N) \rangle$.

The encoding and decoding arrows ε and δ taken together establish a kind of *dictionary*, which allows effective passage from one system to the other and back again. Finally, one must note the extraneous status of the arrows ε and δ, that they are not a part of either systems $\langle N, \kappa(N) \rangle$ or $\langle M, \kappa(M) \rangle$ nor are they entailed by anything in $\langle N, \kappa(N) \rangle$ or in $\langle M, \kappa(M) \rangle$.

Examples and Pluralities

Examples are in order. For instance, Claudius Ptolemy's *Almagest* (*c.* AD 150) contained a brilliant account for the apparent motion of many heavenly bodies. The Ptolemaic system of epicycles and deferents, later with adjustments in terms of eccentricities and equant points, provided good geometric simulations, in the sense that there were enough parameters in defining the circles so that any planetary or stellar trajectory could be represented reasonably accurately by these circular traces in the sky. Despite the fact that Ptolemy did not give any physical reasons why the planets should turn about circles attached to circles in arbitrary positions in the sky, his quantitatively accurate yet qualitatively wrong simulations remained the standard cosmological view for 1400 years. Celestial mechanics has since, of course, been progressively updated with better theories of Copernicus, Kepler, Newton, and Einstein. Each improvement explains more of the underlying principles of motion and not just the trajectories of motion. The universality of the Ptolemaic epicycles is nowadays regarded as an extraneous mathematical artefact irrelevant to the underlying physical situation, and it is for this reason that a representation of trajectories in terms of them can only be regarded as simulation and not as model.

For another example, a lot of the so-called models in the social sciences are really just sophisticated kinds of curve-fitting, i.e., simulations. These activities are akin to the assertion that since a given curve can be approximated by a polynomial, it must be a polynomial. As an illustration, consider that any given set of $n + 1$ functional data points (e.g., hollow dots in Fig. 8) may be fitted *exactly*, with an appropriately chosen set of coefficients $\{a_0, a_1, a_2, \ldots, a_n\}$, onto a polynomial of degree n,

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n. \tag{29}$$

(dashed curve in Fig. 8):

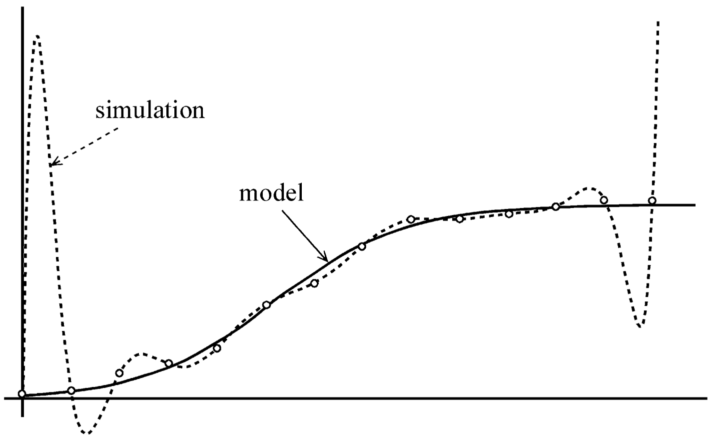


Fig. 8 Simulation versus model

But a more appropriate curve through these points (they in particular being population growth data), is the sigmoid curve

$$y = \frac{K\,P_0\,e^{rt}}{K + P_0(e^{rt} - 1)} \tag{30}$$

(solid curve in Fig. 8). One may note that the solid curve fits the data points less precisely than the dashed curve, and this demonstrates that it is in fact more important for a model to have the functorial correspondence of morphisms (25) than the commutativity (19). Correct functional entailment is the hallmark of a good model, while ‘approximate’ material entailment suffices. Stated otherwise, curve fitting without a theory of the shape of the curve is simulation; model requires understanding of how and why a curve takes its shape. In short:

$$\text{simulation describes; model explains.} \tag{31}$$

‘Simulation’ is based on the Latin word *similis*, ‘like, similar’. A *simulacrum* is ‘something having merely the form or appearance of a certain thing, without possessing its substance or proper qualities’. ‘Model’ in Latin is *modulus*, which means ‘measure’; herein lies a fine nuance that implies a subtle increase in precision. In common usage, however, the two words ‘simulation’ and ‘model’ are often synonyms, meaning (*a*) a simplified description of a system put forward as a basis for theoretical understanding, (*b*) a conceptual or mental representation of a thing, and (*c*) an analogue of a different structure from the system of interest but sharing an important set of functional properties. Some, alternatively, use ‘model’ to mean mathematical theory and ‘simulation’ to mean numerical computation. What I have presented above, however, is how these two words are used in relational biology.

There is a ‘polarity’ built into the general modelling relation of Fig. 7. While both the encoding ε and decoding δ are functors, their operations (and also the roles of the systems $\langle N, \kappa(N)\rangle$ and $\langle M, \kappa(M)\rangle$) are not interchangeable, even for simulations. One reason is that the commutativity $f = \delta \circ g \circ \varepsilon$ of (21) does not imply $g = \varepsilon \circ f \circ \delta$. The issue of when the former implies the latter is a deep topic of investigation, and I shall explicate it elsewhere (Louie 2017).

A special case of congruence between two different natural systems $\langle N_1, \kappa(N_1)\rangle$ and $\langle N_2, \kappa(N_2)\rangle$ occurs when they possess the same formal model $\langle M, \kappa(M)\rangle$ (or alternatively, they constitute distinct realizations of $\langle M, \kappa(M)\rangle$), as shown in Fig. 9.

One readily shows that one can then ‘encode’ the features of $\langle N_1, \kappa(N_1)\rangle$ into corresponding features of $\langle N_2, \kappa(N_2)\rangle$ and, conversely, in such a way that the two entailment structures, in the two systems $\langle N_1, \kappa(N_1)\rangle$ and $\langle N_2, \kappa(N_2)\rangle$, are brought into congruence. That is, one can construct from the above figure a commutative diagram of the form shown in Fig. 10. This is a *mutual* modelling relation between two natural systems (instead of the prototypical unidirectional case from a natural system to a formal one).

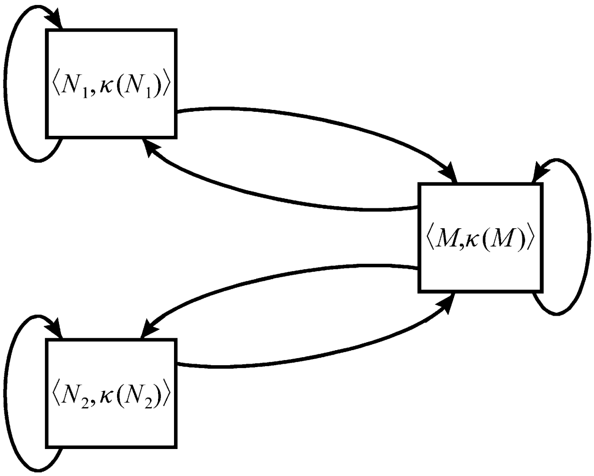


Fig. 9 Common model

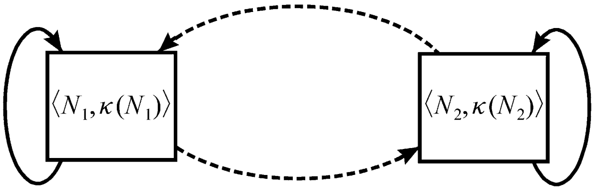
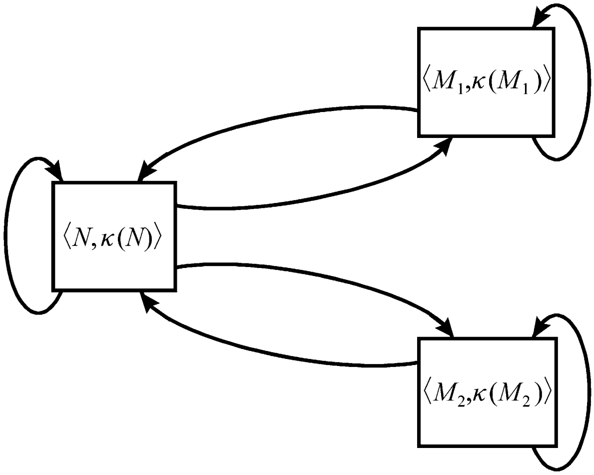


Fig. 10 Analogues

Under these circumstances depicted in the two previous figures, the natural systems $\langle N_1, \kappa(N_1)\rangle$ and $\langle N_2, \kappa(N_2)\rangle$ are *analogues*. Analogous systems allow us to learn about one by observing the other. Relations of analogy underlie the efficacy of 'scale models' in engineering, as well as all of the various 'principles of equivalence' in physics. But the relation of analogy has much deeper consequences. Natural systems of the most diverse kinds (e.g., organisms, societies, economic systems, and political systems) may be analogous. In particular, it is precisely the fact that biological systems and socioeconomic systems are analogous anticipatory systems that provides the unifying foundation and impetus of the present collection of essays that is the *Handbook of Anticipation*. Analogy is a relation between natural systems that arises through the models of their causal entailments, and not directly from their material structures. As such, analogy and its cognates offer a most powerful and physically sound alternative to reductionism (viz., 'share a common model' and therefore 'analogous', as opposed to 'one encompasses the other').

A complementary diagram to that of Fig. 9 is shown in Fig. 11, in which a single natural system $\langle N, \kappa(N)\rangle$ is modelled in two distinct formalisms: $\langle M_1, \kappa(M_1)\rangle$ and $\langle M_2, \kappa(M_2)\rangle$. The question here is: What, if any, is the relation between the formalisms $\langle M_1, \kappa(M_1)\rangle$ and $\langle M_2, \kappa(M_2)\rangle$? The answer here is not in general as straightforward as before; it depends entirely on the extent of the 'overlap' between the two encodings of $\langle N, \kappa(N)\rangle$ in $\langle M_1, \kappa(M_1)\rangle$ and $\langle M_2, \kappa(M_2)\rangle$; i.e., on $\varepsilon_1(N) \cap \varepsilon_2(N) \subset M_1 \cap M_2$ and $\varepsilon_1(\kappa(N)) \cap \varepsilon_2(\kappa(N)) \subset \kappa(M_1) \cap \kappa(M_2)$. In some

Fig. 11 Alternate models



cases, one can effectively build at least some encoding and decoding arrows between the two formalisms. For a couple of examples, consider Dirac's transformation theory formulation of quantum mechanics which unifies Heisenberg's matrix mechanics and Schrödinger's wave mechanics, and the relation between the thermodynamic and statistical-mechanical models of fluids. In other cases, there exists no formal relation between $\langle M_1, \kappa(M_1)\rangle$ and $\langle M_2, \kappa(M_2)\rangle$. One then has the situation in which $\langle N, \kappa(N)\rangle$ *simultaneously* realizes two distinct and independent formalisms; the various Bohr's complementarities for microphysical phenomena are examples.

Encoding and decoding functors may be composed; i.e., one can iteratively model a model. This leads to a second configuration in which a natural system $\langle N, \kappa(N)\rangle$ is related to two formal systems $\langle M_1, \kappa(M_1)\rangle$ and $\langle M_2, \kappa(M_2)\rangle$.

Suppose the formal system $\langle M_1, \kappa(M_1)\rangle$ is a model of the natural system $\langle N, \kappa(N)\rangle$, the prototypical modelling relation being a transition from the realm of science to that of mathematics. If one extracts only the predicative processes of $\langle M_1, \kappa(M_1)\rangle$, one may construct a purely syntactic 'machine' model $\langle M_2, \kappa(M_2)\rangle$ of $\langle M_1, \kappa(M_1)\rangle$, as in Fig. 12.

One may then consider only the outer two systems and forget about the original model $\langle M_1, \kappa(M_1)\rangle$. The formal system $\langle M_2, \kappa(M_2)\rangle$ is a *machine model* of the natural system $\langle N, \kappa(N)\rangle$ and captures the latter's purely syntactic aspects. The encoding and decoding arrows themselves (the dashed arrows in Figure 12) between $\langle N, \kappa(N)\rangle$ and $\langle M_2, \kappa(M_2)\rangle$ cannot be described as *effective* in any formal sense, but they compose exclusively with the input and output strings of the Turing machines in $\langle M_2, \kappa(M_2)\rangle$, and these compositions may immediately be identified with the effective processes in $\langle N, \kappa(N)\rangle$. Whether these exhaust the implicative resources of the system $\langle N, \kappa(N)\rangle$ itself serves to distinguish between *predicative* and *impredicative* systems. (These antonymous adjectives of natural systems are further explicated in the chapter "▶ Complex Systems" in this *Handbook*.)

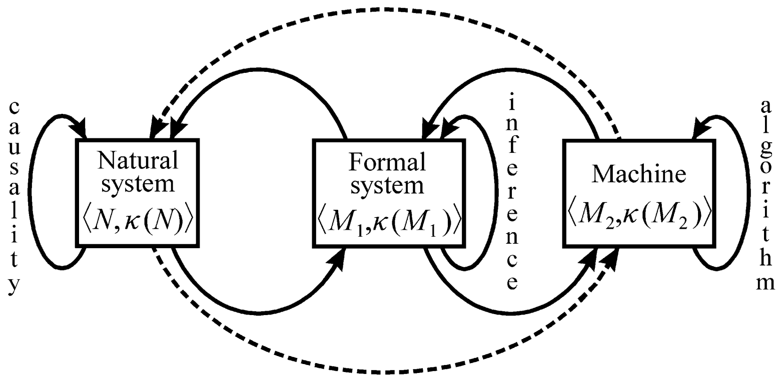


Fig. 12 Machine model

"Internal Predictive Model of Itself and of Its Environment"

Let $\langle S, \kappa(S)\rangle$ be an anticipatory system. The system S partitions the universe U into *self* (S itself) and nonself that is its *environment* ($S^c = U \sim S$). What does S's having a *model of itself and of its environment* mean? 'S itself and its environment' are the whole universe: $S \cup S^c = U$. A model is, however, by necessity incomplete, so it cannot be a model of the 'whole universe' U but only a *proper subsystem* $W \subset U$. Rosen's original definition of anticipatory system actually uses the phrase "of itself and/or of its environment". It is my contention that this construction "and/or" is unnecessary: both "self and/or environment" and "self and environment" describe the universe $U = S \cup S^c$; and the requisite model is a proper subset W of U.

That W is part of 'S itself and its environment' implies it may straddle the self | nonself boundary: so possibly both $W \cap S \neq \varnothing$ and $W \cap S^c \neq \varnothing$. The union $\kappa(W \cap S) \cup \kappa(W \cap S^c)$ of the processes within the two parts does not account for all processes $\kappa(W)$ of W, however. In addition to the internal processes $\kappa(W \cap S) \subset \kappa(S)$ and the environmental processes $\kappa(W \cap S^c) \subset \kappa(S^c)$, $\kappa(W)$ also contains environmental effects on S and how the system S affects its environment. Anticipation in S entails the existence of a model $M \in \mathbf{C}(W)$ and an encoding functor

$$\varepsilon : \langle W, \kappa(W)\rangle \to \langle M, \kappa(M)\rangle. \tag{32}$$

We have already encountered (in the previous section) the multilevel entailments of ε. In particular, one has material entailment

$$\varepsilon : W \to M \tag{33}$$

and functional entailment

$$\varepsilon : \kappa(W) \rightarrow \kappa(M). \tag{34}$$

In common English usage, *predict* means ‘foretell, make a statement about the future’; thus temporal succession is implicit. The word comes from the Latin *prae*, ‘before’, and *dicere*, ‘say’. Note, however, the ‘before’ that the Latin prefix *prae-* (and *pre-*) predicates does not necessarily have to refer to time; it may also be before in place, order, degree, or importance. It is with this general sense that one may distinguish three temporally different classes of ‘predictions’:

(i) *Extenders*, predictions that are time-independent
(ii) *Portents*, predictions that relate simultaneous events
(iii) *Transducers*, predictions that convert information about the world at a given instant into information about the world at some later instant

Time-independent predictions (i) concern a system’s *constitutive parameters*, while time-dependent predictions (ii) and (iii) concern a system’s *dynamics*.

A model M is a *reflector* of its realization W. The functorial images $\varepsilon : W \rightarrow M$ and $\varepsilon : \kappa(W) \rightarrow \kappa(M)$ above all serve to archive a copy of $\langle W, \kappa(W)\rangle$ in $\langle M, \kappa(M)\rangle$. An important purpose of modelling is that through the study of the alternate description $\langle M, \kappa(M)\rangle$, one produces explanations that decode to help in one’s understanding of $\langle W, \kappa(W)\rangle$. A good model should *augur*, i.e., suggest specified outcomes and generate conclusions that are more than the building blocks used in the construction of the model. A model predicts. To whichever class a prediction belongs, what shapes the consequents is not what the encoding ε supplies to the model but, rather, what the decoding δ extracts from the model.

An anticipatory system $\langle S, \kappa(S)\rangle$ has to have more than one inherent dynamics, more than one thing that one may consider ‘time’ (‘real time’ or otherwise). To have anticipation of the system’s own subsequent behavior, something in the system must be running ‘faster than real time’. This is a crucial role that is taken up by the predictive model $\langle M, \kappa(M)\rangle$. The predictive model in an anticipatory system must be able to augur future events; i.e., its predictions must include those belonging to class (iii), transducers. While the *emphasis* is on the requisite “the model’s predictions pertaining to a later instant”, possible roles of $\langle M, \kappa(M)\rangle$ in the other two classes, (i) extenders and (ii) portents, are not excluded. In other words, the model $\langle M, \kappa(M)\rangle$, in addition to offering predictions of potential futures, also considers past and present states, and indeed time-independent aspects, of the system $\langle S, \kappa(S)\rangle$ and its environment. The anticipatory system $\langle S, \kappa(S)\rangle$ then inclusively integrates all these in the execution of its present actions.

One notes that in order to fulfill its purpose of making predictions about the future, the model $\langle M, \kappa(M)\rangle$ must have a ‘faster dynamics’ than its realization $\langle W, \kappa(W)\rangle$. This last phrase is an abbreviation, a terse summary that is interpreted thus: if the trajectories of the system $\langle W, \kappa(W)\rangle$ are parameterized by real time, then the corresponding trajectories of the model $\langle M, \kappa(M)\rangle$ are parameterized by a time variable that goes faster than real time. That is, if $\langle W, \kappa(W)\rangle$ and $\langle M, \kappa(M)\rangle$ both start at time t_0 in equivalent states, and if (real) time runs until $t_1 > t_0$, then $\langle M, \kappa(M)\rangle$

will have proceeded further along its trajectory than $\langle W, \kappa(W)\rangle \cap \langle S, \kappa(S)\rangle$, say to $t_2 > t_1$. This is the sense in which the predictive model $\langle M, \kappa(M)\rangle$ operates on a faster internal time scale than the system $\langle S, \kappa(S)\rangle$ itself. The system $\langle S, \kappa(S)\rangle$ can, therefore, at ‘present time’ t look at the model $\langle M, \kappa(M)\rangle$, which describes the system $\langle W, \kappa(W)\rangle$ at some ‘future time’ $t + h$ (with $h > 0$), and thereby obtain information about its own possible state at $t + h$ if it were to continue on its current trajectory. It is in this way that the behavior of $\langle M, \kappa(M)\rangle$ *predicts* the behavior of $\langle S, \kappa(S)\rangle$. Armed with this information of a possible future, the anticipatory system $\langle S, \kappa(S)\rangle$ may then make trajectory corrections if necessary.

It should be clarified that “anticipation” in Rosen’s usage, embodied in the “predictive” model,does not refer to an ability to ‘see’ or otherwise sense the immediate or the distant future – there is no prescience or psychic phenomena suggested here. Instead, Rosen suggests that there must be information about self, about species, and about the evolutionary environment, encoded into the organization of all living systems. He observes that this *information*, as it behaves through time, is capable of acting causally on the organism’s present behavior, based on relations projected to be applicable in the future. Thus, while not violating time established by external events, organisms seem capable of constructing an internal time scale as part of a model that can indeed be manipulated to produce anticipation. It is in this sense that degrees of freedom in internal models allow time its multi-scaling and reversibility to produce new information. The predictive model in an anticipatory system must not be equivocated to any kind of ‘certainty’ (even probabilistically) about the future. Rosen’s theory of anticipation is a general qualitative theory that describes *all* anticipatory systems. It is not a quantitative theory of *single* systems for which the lore of *large number* of systems, hence statistical reasoning, would ever enter into the picture. In other words, this theory has nothing to do with stochastics. Anticipation is, rather, an assertion based on a model that runs in a faster time scale. The future still has not yet happened: the organism has a *model* of the future but not definitive *knowledge* of future itself. Indeed, the predictive model may sometimes be wrong, the future may unfold very differently from the model’s predictions, and the consequences of the mismatch may be detrimental to the anticipator.

The predictive modelling activity of an anticipatory system is self-contained. That the predictive model is *internal* means

$$\langle M, \kappa(M)\rangle \subset \langle S, \kappa(S)\rangle; \tag{35}$$

that is to say,

$$M \subset S \quad \text{and} \quad \kappa(M) \subset \kappa(S). \tag{36}$$

The encodings (33) and (34) imply

$$\varepsilon(W) \subset M \quad \text{and} \quad \varepsilon(\kappa(W)) \subset \kappa(M). \tag{37}$$

Together with (36), one has

$$\varepsilon(W) \subset S \quad \text{and} \quad \varepsilon(\kappa(W)) \subset \kappa(S). \tag{38}$$

The encodings (33) and (34) also immanently entail the corresponding decoding

$$\delta : M \rightarrow W \tag{39}$$

and

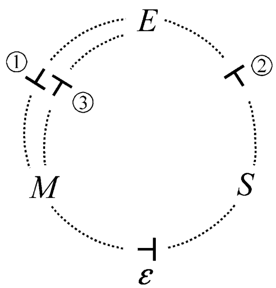
$$\delta : \kappa(M) \rightarrow \kappa(W), \tag{40}$$

whence

$$\delta(M) \subset W \quad \text{and} \quad \delta(\kappa(M)) \subset \kappa(W). \tag{41}$$

These inclusions are succinct summary statements of the embodiment of anticipation, the internal predictive model (Fig. 13).

Now let us return to Fig. 1, the canonical diagram of an anticipatory system. I shall use the same symbols for the object, model, and effector systems, respectively, S, M, and E, to denote their efficient causes. In other words, let each symbol represent the *processor* associated with the block (the "black box") as well as the block itself. Then the entailment diagram for the anticipatory system is



(42)

The mappings labelled with circled numbers correspond to those in Fig. 1. The mapping $\varepsilon : S \rightarrow M$, completing the cycle, is the *encoding* of the object system S into its model M, i.e., the restriction of the encoding functor (32) to $W \cap S$. The hierarchical entailment cycle

$$\{ S \vdash M,\ E \vdash S,\ M \vdash E \} \tag{43}$$

renders this anticipatory system impredicative. (Impredicativity is the defining characteristic of *complex systems* as the latter term is used in relational biology. See the chapters "▶ Relational Biology" and "Complex Systems" in this *Handbook* for further explorations.)

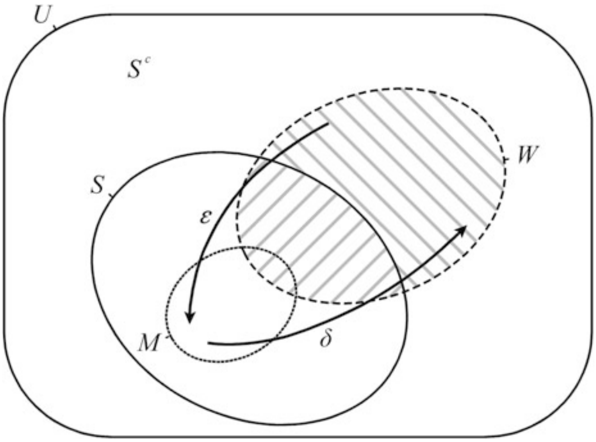


Fig. 13 Internal predictive model of itself and of its environment

"Change ... in Accord with the Model's Predictions"

An anticipatory system, however, has more structure in its entailment pattern than the cyclic permutation of the three maps $\{M, E, S\}$. In particular, the hierarchical action chain

$$S \vdash M \vdash E \vdash S \tag{44}$$

must, as explained above, involve a faster dynamics in M. Also, the set E of effectors functionally entails both the system S and the internal predictive model M:

$$② : E \vdash S \text{ and } ③ : \ E \vdash M, \tag{45}$$

a requisite iterative bifurcation that is not necessarily present in every hierarchical cycle. Thus: "an anticipatory system *must* be complex; a complex system *may* be anticipatory."

Let me fractionate effector E into the functional components E_S that acts on S and E_M that acts on M. While I am at it, I eliminate the circles around the numerical labels of the arrows and split identically numbered arrows into *a*s and *b*s. Recall that the system's environment is $S^c = U \sim S$. After all these modifications, the canonical Fig. 1 of an anticipatory system becomes (Fig. 14).

I reemphasize that what defines an anticipatory system S is not just the *existence* of the internal predictive model – there are *two* indispensable ingredients: (i) internal predictive model M and (ii) *response* E to the prediction. The telos of anticipation is for the system S 'to change state at an instant in accord with the model's predictions pertaining to a later instant'. The central importance of this telos effected by E is reflected in E_S and E_M having the largest number of influent and effluent arrows among the blocks in Fig. 14.

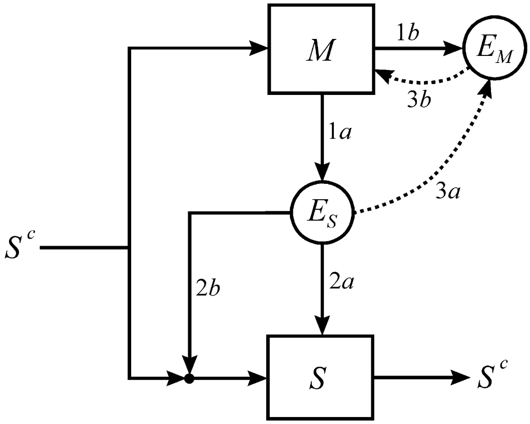


Fig. 14 Anticipatory system with dual effectors

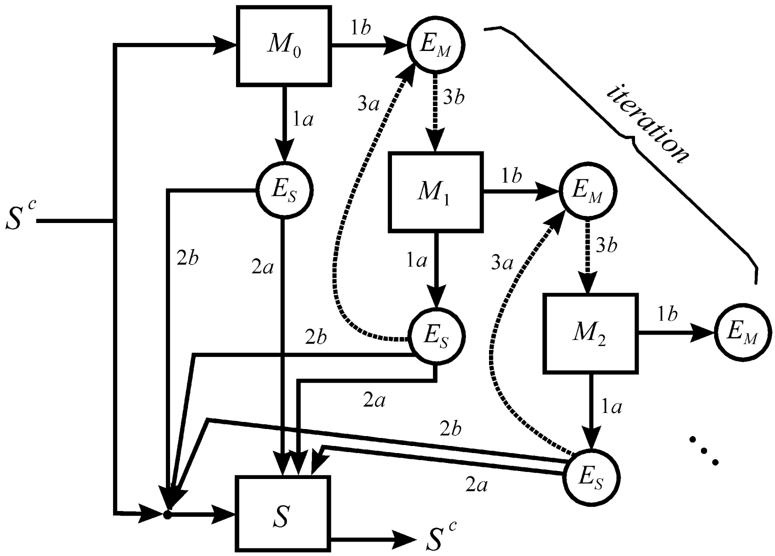


Fig. 15 Anticipatory system with unfolded antecedent actions

That the anticipatory system has to *respond* in accord with the model's prediction means that the control system unit

$$E_M \vdash M \vdash E_S \vdash \tag{46}$$

is *iterated* (for as many times as deemed necessary). With an iteration of cycle (46), the telescoped Fig. 14 unfolds into the form in Fig. 15.

Feedforth

Anticipation is an example of adaptive behavior. Adaptive control system theory is a well-studied subject, but it is mostly formulated in terms of *feedback* control. In a feedback system, control is error-actuated, and the stimulus to corrective action is the discrepancy between the system's actual present state and its targeted state. Stated otherwise, a feedback control system is *reactive*; it must already be departing from its nominal behavior before control begins to be exercised. There are two classes: negative feedback and positive feedback. Negative feedback, the more common variety, involves self-correcting control, with discrepancy-reducing processes that seek to narrow the gap between actual and reference values of a parameter. Stereotypical examples are thermostat, cruise-control system in automobiles, and homeostasis in organisms. Positive feedback, on the other hand, involves self-reinforcing control, with discrepancy-enhancing processes that seek to widen the gap between actual and reference values of a parameter. Many exocrine (enzymatic) and endocrine (hormonal) pathways in living systems engage positive feedback. "Self-fulfilling prophecy" in socioeconomic systems is another example of positive feedback. In most contexts, that feedback is positive or negative is not a value judgment; it does not imply that it correspondingly causes good or bad effects. A negative feedback loop is one that tends to slow down its controlled process, whereas a positive one tends to accelerate it.

Anticipatory control, contrariwise, involves the concept of *feedforth* (or, conventionally but less antonymously termed, "feedforward"). It is through feedforth that the internal predictive model $\langle M, \kappa(M)\rangle$ of an anticipatory system $\langle S, \kappa(S)\rangle$ operates a faster dynamics. Note that feedforth is *not* positive feedback: feedback, whether negative or positive, uses information from the past (discrepancy that is the final cause of processes that have come to pass), while feedforth uses information of the future (as predicted by a model of final causes yet to be entailed). An illustrative example of feedforth is a camera with automatic exposure control. The telos is to set the exposure of a shot such that a relatively constant amount of light will be admitted into the camera, regardless of the ambient light intensity (or, more technically, to adjust the exposure setting to match the mid-tone of the subject to the mid-tone of the image). There are two variables to consider: aperture and shutter speed. (Often the shutter speed is fixed manually, with automatic aperture compensation. But the roles of the two variables may be reversed, or both may be allowed to cooperatively adjust.) It is useless to put a feedback sensor in the camera, for however fast the feedback loop runs, the image will already be over- or underexposed before any corrective measures can be implemented. Rather, an entirely different mode of control is required, manifested in a light meter, which *measures* the ambient light intensity, in conjunction with a *model* that allow the *prediction* of the aperture/shutter speed setting that, for that measured ambient light intensity, will allow an appropriate amount of light into the camera. The camera is then *preset* at this aperture and shutter speed combination before the picture is taken. (The assumption is, of course, that the ambience does not change in the interval between the light-meter's measurement at time t and the actual shot at time $t + h$.) This mode of control, termed *feedforth*, does not involve the propagation of an error signal through the system. It is characterized by the property that *a preemptive action is undertaken, before system performance has deteriorated, on the basis of some predictive model*. This is precisely what is required in an anticipatory system.

Imminence

In this chapter, I have presented:

(i) the *mathematical foundations* of anticipatory systems.

But note that this is neither:

(ii) the mathematical theory of anticipatory systems, nor

(iii) the mathematics of some particular anticipatory system.

The difference between (i) and (ii) is that the former is a metatheory that prescribes the mathematics necessary for the study of anticipation (e.g., the algebra of internal predictive models), while the latter dwells into the mathematical theory of specific tools (e.g., the analysis of control systems as a formal platform of internal predictive models). Stated otherwise, (i) is *about* the requisite ingredients of anticipatory systems and (ii) is the actual study of the ingredients themselves.

There is a cornucopia of chapters in this *Handbook* showing how anticipation specifically arises or is used in a variety of subjects. Occasionally some of these chapters may even contain mathematical tools (iii) that are suitable for the tasks at hand. To proceed from particular instances to the general concept is of course a very common procedure in mathematics. One example, to mention but one analogy, is that 'symmetry' appears abundantly in nature and in every subject of human endeavor; in the minds of mathematicians, the study of symmetry itself is generalized into group theory. In this analogy, when (ii) is group theory, (i) would be the philosophy of symmetry (concepts that need to be accounted for when formulating a comprehensive theory of symmetry, such as harmony, balance, proportion, transformational invariance, etc.), and an example of a specific (iii) may be the algebra of the order $8! \times 3^7 \times (12\,!\,/2) \times 2^{11}$ finite permutation group of Rubik's Cube.

Since anticipatory systems serve as common models for such a diverse collection of scholarly pursuits, it is natural that one would want to have a general mathematical theory (ii) that would lend rigor to the subject. One must not simply linger on stage (i), which is principally a descriptive enterprise of what (ii) has to entail but falls short of being (ii) itself. Indeed, a main purpose for Rosen's publication of *AS* was to move toward such a general theory. The Axiom of Anticipation is a statement of self-evident truth. *AS* provided (along with the philosophical and methodological foundations) the mathematical foundations (i), as explicated and expanded in this chapter, for the study of anticipatory systems. But one must not stagnate on the axiom, use it as a slogan, and rest on its laurels. After proclaiming the obvious, that life is anticipatory, one must move on to the next phase and attempt to develop a mathematical theory of anticipation.

In the 1970s, Rosen lamented that "a study of anticipation is not yet even in its infancy, despite the universality of this mode of control in the biological realm".

There has, alas, not been much progress in the development of a general mathematical theory (ii) in the 40 years thence (neither by Rosen himself nor by others). There is, of course, the tried-and-true dynamical systems (in their various continuous and discrete and recursive and adaptive and, last and decidedly least, computational varieties), but more often than not their invocation in anticipation degenerates into specific applications (iii) instead of being genuine comprehensive theories (ii). In any case, there *ought* to be more to the mathematical tools of anticipation than the less-than-satisfactory standby that are 'glorified and modified dynamical systems'. At the bottom of the barrel, 'computational anticipatory systems' are at best simulations, not models. These two previous sentences are, incidentally, not flippant comments, but are made for good relational-biologic reasons. An anticipatory system must be impredicative, and an impredicative ('complex') system must have at least one nonsimulable ('noncomputable') model. (See the chapter "► Complex Systems" in this *Handbook* for further details.) One must therefore move beyond computation in anticipation.

When *AS* was finally published in 1985, Rosen had added an Appendix (Chapter 7 therein) to briefly sketch the development of the intervening years between 1979 and 1985. It was clear that his interest was not in anticipatory systems themselves, but rather how they had provided a plateau from which to launch the final ascent to the summit that was his one true lifelong quest, the answer to the question "What is life?". Indeed, the Appendix was not so much about anticipation than on complex systems; anticipation was barely mentioned. Rosen's dismissive conclusion was that "our entire treatment of anticipatory systems becomes a corollary of complexity". The enlargement of "this Appendix into a separate monograph in the near future", incidentally, has since been realized as his iconoclastic masterwork *Life Itself* (Rosen 1991). In *Life Itself*, 'anticipation' was again scarcely mentioned in passing; therein, Rosen began Chapter 1 thus:

> This book represents a continuation, an elaboration, and perhaps a culmination of the circle of ideas I have expounded in two previous monographs: *Fundamentals of Measurement and the Representation of Natural Systems* (henceforth abbreviated as *FM*) and *Anticipatory Systems* (abbreviated as *AS*). Both of these, and indeed almost all the rest of my published scientific work, have been driven by a need to understand what it is about organisms that confers upon them their magical characteristics, what it is that sets life apart from other material phenomena in the universe. That is indeed the question of questions: What is life? What is it that enables living things, apparently so moist, fragile, and evanescent, to persist while towering mountains dissolve into dust, and the very continents and oceans dance into oblivion and back? To frame this question requires an almost infinite audacity; to strive to answer it compels an equal humility.

The main conclusion of *Life Itself* is that a natural system is living if and only if it is closed to efficient causation, a property which in particular renders a living system impredicative ('complex') (*cf.* the chapter "Relational Biology" in this *Handbook*). After *AS*, Rosen never worked on anticipatory systems themselves again; a couple of on-topic post-*AS* publications constituted recompiled, relinked, and otherwise rearranged efforts, but no further developments. (Anticipatory system concepts would of course be new to readers who had not previously encountered them, but these regurgitations did not contain anything new in the relational-biologic canon.) When anticipation was mentioned at all in the rare instances, they were invariably, so Rosen had declared, as “a corollary of complexity”.

It is important to remember that both complexity (impredicativity) and anticipation are necessary conditions for life, the containment hierarchy being

$$\text{Impredicativity} \supset \text{Anticipation} \supset \text{Life}. \tag{47}$$

While we in relational biology may find it more congenial to characterize life in terms of impredicativity instead of anticipation, this does not in any way diminish the indispensability of anticipation in the understanding of biological, human, and social sciences. There are deep system-theoretic homologies among these sciences. Analogy allows the possibility of obtaining insights into anticipatory processes in the human and social realms from the understanding of biological anticipation. To this end, a comprehensive general mathematical theory of anticipatory systems is the means. This is a quest, however quixotic a journey it may seem to be, we must continue.

Summary

$$\text{Life} \subset \textbf{Anticipation} \subset \text{Impredicativity}$$

Life anticipates. Social, human, and many other natural systems also anticipate. The behaviors of anticipatory systems are largely determined by the nature of their internal predictive models. This chapter lays the mathematical foundations for the study of this important class of model-based systems. I leave the last words to our founder; Robert Rosen closed (the 1979 first draft of) *AS* thus:

> The study of anticipatory systems thus involves in an essential way the subjective notions of good and ill, as they manifest themselves in the models which shape our behavior. For in a profound sense, the study of models is the study of man; and if we can agree about our models, we can agree about everything else.

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