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# Relational biology and Church's thesis

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ABSTRACT

Church's Thesis is a metamathematical hypothesis that says the concepts of effective calculability and computability are coextensive. It is reasonable to consider everything that happens in the material world to be 'effective'. If Church's Thesis were true in the natural world, then it would mean that all material processes could be expressed in purely syntactic terms. A corollary in relational biology is that a living system must have non-computable models. Thus the existence of living systems implies that Church's Thesis is false as a physical proposition. The paper begins with a review of the tenets of relational biology, which is the standpoint from which this exposition on the Foundations of Mathematics and Theoretical Biology is composed.

## 1. Praefatio

I begin with a summary of the premise of this paper. The remaining sections, 2–7, are then my attempt to make the following sequence of statements, 1.1–1.11, intelligible to the reader. The exposition is necessarily concise and unavoidably incomplete. For further exploration of the topics discussed, the reader is cordially invited to consult the three books I have (so far) written on the subject of relational biology:

*More Than Life Itself: A Synthetic Continuation in Relational Biology*
(Louie, 2009);

*The Reflection of Life: Functional Entailment and Imminence in Relational Biology*
(Louie, 2013);

*Intangible Life: Functorial Connections in Relational Biology*
(Louie, 2017b).

Category theory is the metalanguage of relational biology. The definitive reference is Mac Lane (1978). I must suppose that the reader has some familiarity with this branch of abstract algebra, otherwise the present paper would be overwhelmed by 'an introduction to category theory' before proceeding. A summary of those category-theoretic concepts that appear henceforth may also be found in the Appendix of *More Than Life Itself* (Louie, 2009, *pp*. 329–372) and the Prolegomenon of *Intangible Life* (Louie, 2017b, *pp*. 1–24).

The following sequence of statements connects Relational Biology to Church's Thesis.

### *1.1. Definition*

A natural system is *closed to efficient causation* ('*clef*') if its every efficient cause is entailed within the system.

### *1.2. Fundamental theorem of relational biology*

*A natural system is an organism* (*a living system*) *if and only if it is closed to efficient causation.*

### *1.3. Theorem*

*The following two properties of a natural system are equivalent:*

(a) *its every efficient cause is entailed within the system;*
(b) *it has a model that has all its processes contained in hierarchical cycles.*

Therefore, one has the

### *1.4. Theorem*

*A natural system is clef if and only if it has a model that has all its processes contained in hierarchical* cycles.

A hierarchical cycle is not Turing-computable; consequently:

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*1.5. Theorem*

*If a formal system contains a hierarchical cycle, then it is nonsimulable* (*not Turing-computable*).

*1.6. Corollary*

*A clef system has a nonsimulable model.*

*1.7. Corollary*

*A living system has a nonsimulable model.*

*1.8. Common notion*

Everything that happens in the material world must be considered "*effective*".

One thus has the syllogism:

*1.9. Rosen's theorem*

*The nonsimulable model of a living system is effective but not computable.*

*1.10. Church's thesis (terse form)*

*Effective means computable.*

*1.11. ⊨ Conclusion*

*The existence of living systems implies that Church's Thesis is* false *as a physical proposition.*

## 2. System

'*System*' is a primitive. It takes on the intuitive meaning:

*2.1. Definition*

A *system* is a collection of material or immaterial things that comprises one's subject of study.

When the 'subject of study' is in the external world of sensory phenomena with their causal entailment, the system is called a natural system. When the object of study is the internal world of ideas with their inferential entailment expressed in some language, the system is called a formal system.

*2.2. Definition*

A *material system* is a physical object in the world.

*2.3. Definition*

A *natural system* is

i. a part, whence a subset, of the external world; and
ii. a collection of qualities, to which definite relations can be imputed.

There is a subtle difference between a material system (also called a physical system or a physicochemical system) and a natural system. A material system is ontological, it being simply *any* physical object in the world. A natural system, on the other hand, is epistemological, since the partitioning of the external world and the formation of percepts and their relations are all mental constructs (and are therefore entailed by the bounds of mental constructs). In short, a *natural system is a subjectively-defined representation of a material system*. In a natural system that comprises one's subject of study, the material things are the physical components, and the immaterial things are the processes that are causal interactions among the components.

*2.4. Definition*

A *formal system* is an object in the universe of mathematics.

'An object in the universe of mathematics' may be interpreted as an appropriately defined category $\mathbf{C}$. A category $\mathbf{C}$ comprises of two collections:

i. *Objects* $\mathcal{O}\mathbf{C}$, and
ii. *Morphisms* $\mathcal{A}\mathbf{C}$.

In a formal system that comprises one's subject of study, the objects $\mathcal{O}\mathbf{C}$ are the collection of 'material things'; and the morphisms $\mathcal{A}\mathbf{C}$ are the collection of 'immaterial things', the processes that are inferential interactions among the objects, i.e., the mappings. Therefore, one has the alternative

*2.5. Definition*

A *formal system* is a category $\mathbf{C} = \langle \mathcal{O}\mathbf{C}, \mathcal{A}\mathbf{C} \rangle$, comprising a collection $\mathcal{O}\mathbf{C}$ of objects and a collection $\mathcal{A}\mathbf{C}$ of morphisms (alternatively called *arrows*, hence the symbol '$\mathcal{A}$', satisfying the requisite category-theoretic axioms.

It is sufficient for our purpose to assume that an object $X \in \mathcal{O}\mathbf{C}$ is a *set*, and a morphism $f \in \mathcal{A}\mathbf{C}$ is a *mapping* with as domain one such object $X$ and as codomain one such object $Y$; i.e., $f \in \mathbf{C}(X, Y)$ for $X, Y \in \mathcal{O}\mathbf{C}$, alternatively denoted $f: X \to Y$. The collection $\mathbf{C}(X, Y) \subset \mathcal{A}\mathbf{C}$ of $\mathbf{C}$-morphisms with domain $X$ and codomain $Y$ is called a *hom-set*.

*2.6. Definition*

For a mapping $f: X \to Y$, let $a \in X$, then $f(a) = b \in Y$. The input element $a$ is the *material cause* of the mapping; the processor $f$ itself is 'that which entails' and is the *efficient cause* of the mapping; the output element $b$ is 'that which is entailed by $f$' and is the *final cause* of the mapping. This situation may be denoted $f: a \mapsto b$ as well as $f \vdash b$, and the formal structure of this entailment process is the *formal cause* of the mapping.

For a detailed exposition of the four Aristotelian causes and their manifestations in a mapping, see Chapter 5 of Louie (2009) and Chapter 6 of Louie (2013).

One of the properties that morphisms must satisfy is that they *compose*: if $f: X \to Y$ and $g: Y \to Z$, then their (*sequential*) *composite* $g \circ f \in \mathcal{A}\mathbf{C}$ is such that $g \circ f: X \to Z$. In $f: a \mapsto b$ and $g: b \mapsto c$, the final cause $b$ of $f$ is relayed as the material cause of the mapping $g$. In this case, when $f \vdash b$ and that which is entailed is a material cause (of another mapping), one has the

*2.7. Definition*

The entailment of a material cause is called *material entailment*.

It may happen that a hom-set of a category $\mathbf{C}$ is itself a $\mathbf{C}$-object; e.g., $Y, Z \in \mathcal{O}\mathbf{C}$ and $\mathbf{C}(Y, Z) \in \mathcal{O}\mathbf{C}$. Then a mapping $f: X \to \mathbf{C}(Y, Z)$ is such that for $a \in X, f(a) = g: Y \to Z$. In $f: a \mapsto g$, the final cause $g$ of $f$ is *itself* an efficient cause (of the mapping $g \in \mathcal{A}\mathbf{C}$). In this case, when $f \vdash g$ and that which is entailed is an efficient cause, one has the

*2.8. Definition*

The entailment of an efficient cause is called functional *entailment*.

Since the codomain $\mathbf{C}(Y,\ Z)$ of $f$ contains $g$ (i.e., $f \in \mathbf{C}(X,\ \mathbf{C}(Y,\ Z))$ and $g \in \mathbf{C}(Y,\ Z)$), the mapping $f$ may be considered to occupy a higher 'hierarchical level' than the mapping $g$. Thus an iteration of 'efficient cause of efficient cause' is inherently hierarchical. The entailment $f \vdash g$ is called *hierarchical composition*.

In any system, whether formal or natural, the material and immaterial things, i.e., the material and efficient causes, must be characterized together. Thus, extending the $\langle \mathcal{O}\mathbf{C},\ \mathcal{A}\mathbf{C} \rangle$ notation for a formal system, a system (either natural or formal) may be represented as $\langle S,\ \kappa(S) \rangle$ where $S$ is the collection of all material causes (i.e., objects) and $\kappa(S)$ is the collection of all efficient causes (i.e., morphisms). The pairs $\langle S,\ \kappa(S) \rangle$ are variously manifested as

(a) objects and morphisms;
(b) states and observables;
(c) structure and function;
(d) material and functional entailments;
(e) sequential and hierarchical composites;
(f) metabolism and repair;

etc.

Just as mappings and their compositions in formal systems, causal processes in a natural system similarly connect their interacting components. On this level of generality, then, a system in relational, hence non-material, terms may therefore transcend the temporary (and temporal) objects and be considered simply as a *network of interacting processes*, where each process is either a material entailment or an efficient entailment. The network, when represented in graph-theoretic form, is an arrow diagram.

The entailment network of a system may give rise to a very complicated pattern of interconnecting material and functional entailments. When several mappings are linked by sequential compositions, one has a *sequential chain*. When the first and last mappings in a sequential chain are themselves linked by sequential composition, the chain folds up into a *sequential cycle*. For example, a three-mapping sequential cycle may be $f: X \to Y$, $g: Y \to Z$, and $h: Z \to X$, with $f: a \mapsto b$, $g: b \mapsto c$, and $h: c \mapsto a$. A sequential cycle is a closed path of material causation.

Likewise, hierarchical compositions may form a *hierarchical chain*: $f_1 \vdash f_2 \vdash \cdots \vdash f_n$; and when $f_n = f_1$ a hierarchical cycle. For example, a three-mapping *hierarchical cycle* may be $f \vdash g \vdash h \vdash f$, where $f \in \mathbf{C}(W,\ \mathbf{C}(X,\ \mathbf{C}(Y,\ Z)))$, $g \in \mathbf{C}(X,\ \mathbf{C}(Y,\ Z))$, and $h \in \mathbf{C}(Y,\ Z)$ such that $Z \cong \mathbf{C}(W,\ \mathbf{C}(X,\ \mathbf{C}(Y,\ Z)))$. A hierarchical cycle is a *closed path of efficient causation*. *Both* the hierarchy and the cycle (closed loop) are essential attributes of this closure.

The existence of hierarchical cycles in an entailment network turns out to be the key partitioning characteristic of systems.

*2.9. Partition*

A system $S$ contains either

I. *no* hierarchical cycles (i.e., *none* of the efficient causes in $S$ is involved in a hierarchical cycle); or
II. some (i.e., one or more) hierarchical cycles.

Let $P$ be the proportion of efficient causes $f \in \kappa(S)$, among all efficient causes $\kappa(S)$ of $S$, that are involved in a hierarchical cycle. Then for systems of class I, $P = 0$, and for systems of class II, $0 < P \leq 1$. Systems of the extreme case when $P = 1$ (i.e., when *all* of the efficient causes in $S$ are involved in hierarchical cycles) are decidedly special, and are given a specific name:

*2.10. Definition*

A system is *closed to efficient causation* (= '*clef*') if its every efficient cause is entailed within the system, i.e., if every efficient cause is functionally entailed within the system.

*2.11. Theorem*

*The following two properties of a natural system are equivalent:*

(a) *its every efficient cause is entailed within the system;*
(b) *it has a model that has all its processes contained in hierarchical cycles.*

Therefore, clef systems are precisely systems of the extreme case when $P = 1$:

*2.12. Theorem*

*A natural system is clef if and only if it has a model that has all its processes contained in hierarchical cycles.*

Simply from the probability distribution of random numbers $P$ in the interval $[0,\ 1]$, one may arrive at the conclusion that class I ($P = 0$) and clef ($P = 1$) systems are rare ("meagre"), while almost all systems belong to class II ($0 < P \leq 1$). I shall have more to say about this partitioning of systems in Section 5 below.

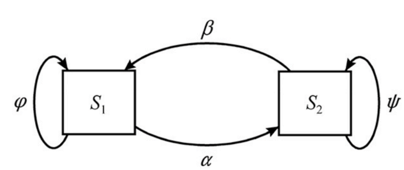
It is important to note that 'closure to efficient causation' is a condition on efficient causes, *not* on material causes. Thus a natural system that is closed to efficient causation is not necessarily a 'closed system' in the thermodynamic sense. (In thermodynamics, a *closed system* is one that is *closed to material causation*, i.e., a system that allows energy but not matter to be exchanged across its boundary.)

If a system $\langle S,\ \kappa(S) \rangle$ is closed to efficient causation, one may symbolically write

$$\forall f \in \kappa(S) \quad \exists\, \Phi \in \kappa(S): \quad \Phi \ \vdash f. \tag{1}$$

## 3. The modelling relation

When there are two systems, where each system can be either natural or formal, they invite comparison.




(2)

The purpose of the comparison is that one may learn something new about a system $S_1$ of interest by studying a different system $S_2$ that is its *surrogate*. The arrows $\varphi$ and $\psi$ represent entailment within, respectively, the systems $S_1$ and $S_2$, and as such are *intra*-system processes *internal* to their own systems.

The arrow $\alpha$ serves to associate features of $S_1$ with their counterparts in $S_2$, while the arrow $\beta$ serves to associate features of $S_2$ with those of $S_1$. The arrows $\alpha$ and $\beta$ taken together thus establish a kind of *dictionary*, which allows effective passage from one system to the other and vice versa. The arrows $\alpha$ and $\beta$ are *external* to both $S_1$ and $S_2$. These *inter*-system processes are not a part of, nor are they entailed by anything in, either systems.

Since the arrows $\varphi$ and $\psi$ are internal and therefore inherent to their own systems, the vehicle for establishing a relation of any kind between $S_1$ and $S_2$ resides in the choice of the external arrows $\alpha$ and $\beta$. So far the relation $\langle S_1, \varphi, \alpha \rangle \leftrightarrow \langle S_2, \psi, \beta \rangle$ is symmetrical. Symmetry breaks when one system is reflected in the other.

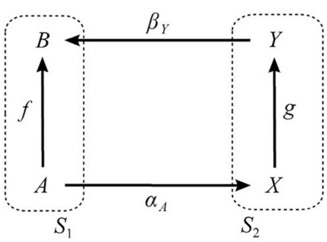
### 3.1. Simulation

A *necessary condition* for reflection involves all four arrows, and may be stated as 'whether one follows *path* $\varphi$ or *paths* $\alpha,\ \psi,\ \beta$ *in sequence*, one reaches the same destination'. Formally, this may be expressed as the compositional equality

$$\varphi \ = \ \beta \circ \psi \circ \alpha. \tag{3}$$

If this relation is satisfied, one says that $S_2$ is a *simulation* of $S_1$, that $\alpha : S_1 \to S_2$ is the *encoding* arrow, and $\beta : S_2 \to S_1$ is the *decoding* arrow. The reflection entails a chirality, that of an 'original' and its 'image', since evidently equation (3) is not the same as $\psi \ = \alpha \circ \varphi \circ \beta$; the truths of the two equality statements are independent.

Equation (3) is an abbreviation. Let $f$ be a process in the entailment structure of the arrow $\varphi$ in system $S_1$. It is more convenient for exposition to represent this process, be it causal or inferential entailment, as a mapping $f : A \to B$ (whence the sets $A$ and $B$ represent component objects in $S_1$). The encoding $\alpha : S_1 \to S_2$ of systems at the component $A$ hierarchically entails the element mapping $\alpha_A : A \to X$, where $X$ is a component object in $S_2$. Consider a mapping $g : X \to Y$ (which is a process in the entailment structure of the arrow $\psi$, and where the sets $X$ and $Y$ represent component objects, in $S_2$) and the hierarchical action of the decoding $\beta : S_2 \to S_1$ which in turn entails the element mapping $\beta_Y : Y \to B$. Suppose the four mappings connect to make the diagram




(4)

*commute*. This means for every element $a$ in $A$, whether one traces through the mapping $f$ alone, or through $\alpha_A$ followed by $g$ and then followed by $\beta_Y$, one gets the same result in $B$; i.e. *for all* $a \in A$ the equality

$$f(a) \quad = \quad \beta_Y(\ g(\ \alpha_A(a)\ )\ ) \tag{5}$$

holds. Note that this commutativity condition for simulation places no further restrictions on the mapping $g$ itself and its domain $X$ and codomain $Y$, other than that the relay that begins at $a \in A$, of the composite trace

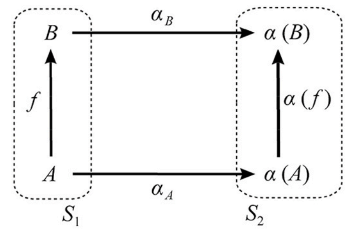
$$a \ \longmapsto \ \alpha_A(a) \longmapsto g(\ \alpha_A(a)\ ) \longmapsto \beta_Y(\ g(\ \alpha_A(a)\ )\ ) \ = \ b, \tag{6}$$

needs to reach the correct final destination $b = f(a) \in B$ . Such emphasis on the results *regardless of the manner in which they are generated* (i.e. with no particular concern on underlying principles) is the case when $S_2$ is a *simulation* of $S_1$.

The idea of simulation is irrevocably tied to the requirement for an extraneous *machine* (the mapping $g : X \to Y$) that implements it: $g$ in $S_2$ is something that has no counterpart — it is neither encoded from nor decoded into anything — in the system $S_1$ being simulated. Simulation, in other words, depends on mathematical relationships that are, from the standpoint of what is simulated, *artefactual*. Consequently, the causal structure of the system $S_1$ being simulated is not preserved in the simulation $S_2$.

### 3.2. Model

If, however, the mapping $g$ is *itself* entailed by the encoding $\alpha$, i.e. if $g \ = \alpha(f)$, whence the mapping in $S_2$ is $\alpha(f) : \alpha(A) \to \alpha(B)$, then one has the commutative diagram

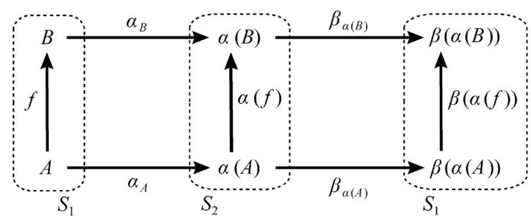



(7)

which encompasses the equality that says, for every element $a \in A$,

$$\alpha_B(\ f(a)\ ) \quad = \quad \alpha(f)(\ \alpha_A(a)\ ) \quad \in \quad \alpha(B). \tag{8}$$

Further, the decoding $\beta$ has to suitably invert the process, so that $\alpha(f) : \alpha(A) \to \alpha(B)$ gets mapped back to a process $\beta(\alpha(f)) : \beta(\alpha(A)) \to \beta(\alpha(B))$ in $S_1$, such that one has the composite commutative diagram




(9)

which contains, in addition to (8), the equalities

$$\begin{aligned} &\beta_{\alpha(B)}(\ \alpha_B(f(a))\ ) \quad = \quad \beta_{\alpha(B)}(\ \alpha(f)(\alpha_A(a))\ ) \\ &= \quad \beta(\alpha(f))\big(\ \beta_{\alpha(A)}(\alpha_A(a))\ \big) \quad \in \quad \beta(\alpha(B))\ . \end{aligned} \tag{10}$$

Equalities (10) encompass three relays:

$$a \longmapsto \left\{ \begin{array}{c} f(a) \longmapsto \alpha_B(f(a)\ ) \longmapsto \beta_{\alpha(B)}(\alpha_B(f(a)\ )\ ) \\ \alpha_A(a) \longmapsto \ \alpha(f)(\alpha_A(a)\ ) \longmapsto \beta_{\alpha(B)}(\alpha(f)(\alpha_A(a)\ )\ ) \\ \alpha_A(a) \longmapsto \beta_{\alpha(A)}(\alpha_A(a)\ ) \longmapsto \beta(\alpha(f)\ )\big(\beta_{\alpha(A)}(\alpha_A(a)\ )\ \big) \end{array} \right\} \quad = \quad b, \tag{11}$$

and say that all three compositional paths have to reach the same element in $b \in \beta(\alpha(B))$. Further, the process $\beta(\alpha(f)) : \beta(\alpha(A)) \to \beta(\alpha(B))$ must be, in an appropriately defined sense, comparable to the original process $f : A \to B$ in $S_1$, such that the congruences

$$\left\{ \begin{array}{ccc} \beta(\alpha(A)) & \cong & A \\ \beta(\alpha(B)) & \cong & B \\ \beta(\alpha(f)) & \cong & f \end{array} \right. \tag{12}$$

hold. When these more stringent conditions are satisfied, the simulation is called a *model*. If this *modelling relation* is satisfied between the systems $S_1$ and $S_2$, one then says that there is a *congruence* between their entailment structures, that $S_2$ is a *model* of $S_1$, and that $S_1$ is a *realization* of $S_2$.

$$\textbf{Realization} \ \underset{\textit{Encoding}}{\overset{\textit{Decoding}}{\rightleftarrows}} \ \textbf{Model} \tag{13}$$

This kind of congruence between entailment structures is, of course, defined by the category-theoretic entity called *functor*. Indeed, the commutative diagram (7) is the very picture of a functor. The encoding $\alpha$ has a functorial representation in

$$\alpha \ : \ \left\{ \begin{array}{cc} A \longmapsto \alpha A & (A \in \{S_1 \ \text{objects}\}\ ) \\ [f : A \to B] \longmapsto [\alpha f : \alpha A \to \alpha B] & (f \in \{S_1 \ \text{processes}\}\ )\ , \end{array} \right. \tag{14}$$

and the decoding functor $\beta$ has a slightly more complicated representation in

$$\beta \ : \ \begin{cases} \alpha A \longmapsto \beta(\alpha A) \quad (\alpha A \ \in \ \{\alpha - \text{encoded } S_2 \text{ objects}\}) \\ [\alpha f : \alpha A \to \alpha B] \longmapsto [\beta(\alpha f) : \beta(\alpha A) \to \beta(\alpha B)] \\ \qquad (\alpha f \ \in \ \{\alpha - \text{encoded } S_2 \text{ processes}\}) \ . \end{cases} \tag{15}$$

While both models and simulations are manifestations of the commutativity of diagram (2), a simulation of a process just provides an alternate description of the entailed effects, whereas a model is a special kind of simulation that additionally also provides an alternate description of the entailment structure of the mapping representing the process itself. In a model $S_2$ of $S_1$, the causal structure of $S_1$ must be faithfully preserved. It is, therefore, easier to obtain a simulation than a model of a process; compare the single relay (6) with the triple relay (11).

Examples are in order. For instance, Claudius Ptolemy's *Almagest* (*c*. AD 150 CE) contained an account for the apparent motion of many heavenly bodies. The Ptolemaic system of epicycles and deferents, later with adjustments in terms of eccentricities and equant points, provided good geometric simulations, in the sense that there were enough parameters in defining the circles so that any planetary or stellar trajectory could be represented reasonably accurately by these circular traces in the sky. Despite the fact that Ptolemy did not give any physical reasons why the planets should turn about circles attached to circles in arbitrary positions in the sky, his simulations remained the standard cosmological view for 1400 years. Celestial mechanics has since, of course, been progressively updated with better theories of Copernicus, Kepler, Newton, and Einstein. Each improvement explains more of the underlying principles of motion, and not just the trajectories of motion. Any planetary orbit may be approximated to any desired degree of accuracy with epicycles, because a smooth curve can be approximated with enough circular arcs. That epicycles work is not because of a physical property, but because it is a consequence of a mathematical principle, which, indeed, has no physical counterpart. The universality of the Ptolemaic epicycles is, stated otherwise, an extraneous mathematical artefact irrelevant to the underlying physical situation, and it is for this reason that a representation of trajectories in terms of them can only be regarded as simulation, and not as model.

As another example, a lot of the so-called 'models' in the social sciences are really just sophisticated kinds of curve-fitting, i.e. simulations. These activities are akin to the assertion that since a given curve can be approximated by a polynomial, it must be a polynomial. Stated otherwise, curve-fitting without a theory of the shape of the curve is simulation; model requires understanding of how and why a curve takes its shape. I refer the reader to Louie (2017a) for further examples and for a more detailed discussion on the modelling relation.

In common usage, the two words 'simulation' and 'model' are often synonyms, meaning:

- a simplified description of a system put forward as a basis for theoretical understanding
- a conceptual or mental representation of a thing
- an analog of different structure from the system of interest but sharing an important set of functional properties.

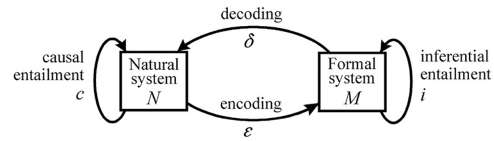
Some, alternatively, use 'model' to mean mathematical theory, and 'simulation' to mean numerical computation. What I have presented above are the meanings of these two words *as they are used in relational biology*.

### 3.3. Ad summam

Simulation describes; model explains.

### 3.4. The canon

A *modelling relation* is thus a commutative functorial encoding and decoding from one system to another. From a natural system $N$ to a formal system $M$, the situation may be represented in the following canonical diagram:



(16)

(I have replaced the symbols $\langle S_1, \ \varphi, \ \alpha \rangle \to \langle S_2, \ \psi, \ \beta \rangle$ in the generic diagram (2) with the specific $\langle N, \ c, \ \varepsilon \rangle \to \langle M, \ i, \ \delta \rangle$.) Any system is characterized by the entailments within. In natural systems, these entailments are governed by causality $c$. Causality is the principle that every effect has a cause and is a reflection of the belief that successions of events in the world are governed by definite relations. In formal systems, entailments take the form of inferences $i$ governed by explicit production rules. The encoding $\varepsilon$ maps the natural system $N$ and its causal entailment $c$ therein to the model formal system $M$ and its internal inferential entailment $i$; i.e., $\varepsilon$ acts functorially on objects and processes thus:

$$\varepsilon \ : \ \begin{cases} N \ \to \ M \\ \ c \ \to \ i \end{cases}. \tag{17}$$

The decoding $\delta$ does the reverse on the encoded objects and processes, mapping the formal system $\varepsilon(N) \subset M$ to its realization $\delta(\varepsilon(N)) \subset N$:

$$\delta \ : \ \begin{cases} \varepsilon(N) \ \to \ \delta(\varepsilon(N)) \\ \ \varepsilon(c) \ \to \ \delta(\varepsilon(c)) \end{cases}. \tag{18}$$

The commutativity condition (3), when translated into these initial-predicated notations, becomes

$$c \ = \ \delta \circ i \circ \varepsilon, \tag{19}$$

which says that tracing through the causal entailment arrow $c$ is the same as tracing successively through the three arrows, $\varepsilon$ncoding $\varepsilon$, *i*nferential entailment $i$, and $\delta$ecoding $\delta$.

### 3.5. Natural Law

That the two modes of entailment, causality $c$ and inferences $i$, can be brought into a congruence in such a way that inference in a formalism mirrors causality in the world and conversely, explicitly embodied in a modelling relation between a natural system and a formalism, is an article of faith in science commonly known as *Natural Law*. Natural order is woven into the fabric of reality. Natural Law posits the existence of these entailment relations and that this causal order can be imaged by implicative order. In terms of the modelling relation (16), Natural Law is an *existential declaration* of causal entailment $c$ and the encodings $\varepsilon : N \mapsto M$ and $\varepsilon : c \mapsto i$.

The very fact that the right-hand side of the modelling relation diagram (16) is an entity $\langle M, \ \kappa(M) \rangle$ in the universe of mathematics implies, in a sense, that the concept of Natural Law entails the efficacy of mathematics. One must recognize, however, the fact that natural systems from the external world generate in us our percepts of 'reality'. While causal entailments *themselves* may be universal truths, *perceived* causal entailments are *not*. All we have are our own *observations*, *opinions*, *interpretations*, our individual *alternate descriptions* of 'reality' that are our personal *models* of 'truth'. Causal entailments are interpreted, not proven.

A mathematical proof is absolute; it is categorically more than a scientific 'proof' (and a judicial 'proof') of 'beyond a reasonable doubt'. A scientific theory can never be proven to the same absolute certainty of a mathematical theorem. This is because a scientific 'proof' is merely considered 'highly likely based on the evidence available'; it depends on observation, experimentation, perception, and interpretation — all of which are fallible, and are in any case approximations of truth. Sometimes the minimized doubt later turns out to be errors, and paradoxically, in the same spirit of 'errors drive evolution', the inherent weakness in scientific proofs leads to scientific revolutions, when, 'based on new evidence', 'proven' theories are refined, updated, surpassed, or replaced. One cannot 'prove a negative' in science: a failure to observe a phenomenon thus far does not mean that it will never be observed, whence does not imply its nonexistence. But one can certainly, indeed routinely, prove a negative in mathematics, that something does not (and cannot) exist.

Modelling is the art of bringing entailment structures into congruence. The essence of an art is that it rests on the heuristic, the unentailed, the intuitive leap. The encoding and the decoding arrows in the modelling relation diagram are themselves unentailed. Natural Law assures the modellers only that their art is not in vain, but it in itself provides not the slightest clue how to go about it. There is no right or wrong in art, only similarities and differences, the congenial and the uncongenial.

Among the four arrows in the diagram of the modelling relation, only inferential entailment $i$ may be *proven* in the rigorous mathematical sense. Absolute statements about the truth of statements validated by proofs cannot be disputed. The other three arrows in the modelling relation diagram — causal entailment $c$, encoding $\varepsilon$, and decoding $\delta$ — all have intuitive elements. As such, one if so inclined may claim that another's interpretations are *uncongenial*, but cannot conclude that they are *wrong*: there are no *absolute truths* concerning them. After all, modelling itself, the choice of appropriate sets and mappings $\langle \varepsilon, M, \delta \rangle$ of encoding, formal system, and decoding given a natural system $N$, is more an art than a science. A model is, by definition, almost always incomplete.

## 4. Relational biology

Relational biology is a study of life in terms of the organization of entailment relations in living systems, independent of any particular physical mechanism or material realization.

### 4.1. Nicolas Rashevsky

(1899–1972) developed the strategy of *relational biology* to capture both the integrative aspects of individual organisms and the unity of the living world. A motivation is the concept of the *universality* of a set of analytic units into which an organism is to be decomposed. These analytic units, the 'elements' in relational biology, are behaviours; contrast this with the reductionist strategy in which the elements are structural subunits.

The idea of universality of behaviour must begin with the acknowledgment of our fundamental intuition that all organisms are in some sense *similar* to one another, and dissimilar to nonliving systems. The basis of biology itself rests on the fact that living systems share common characteristics that are absent (or at least incomplete) in the nonliving. The precise articulation of these characteristics had, however, been elusive; one could not satisfactorily define "life", particularly so if one insisted on attempting to formulate such definitions entirely in structural terms. Rashevsky thus (in the 1950s) concluded that whatever the specific characteristics of organisms might be, they had to involve *relations* rather than structures. An organism is whence an entailment network of its processes; the universal analytic units in the decompositions are mappings, the nature of relational interactions among which being what characterizes a living system. Stated otherwise, integrated homological behaviours of organisms involve

(R1) *biological processes* (i.e., *biological functions* that are metabolism, respiration, reproduction, repair, etc.),
(R2) *relations* connecting these biological processes, and
(R3) *interactions* of these processes with the environment.

Properties of the whole *emerge* from the *integration* of (R1), (R2), and (R3). *A fortiori*, analysis of a decomposition of a biological system into functional components (R1) themselves, i.e., knowledge of the components alone, tells a limited, partial, and deficient story.

The strategy of relational biology is, in sum, to study biology as organization of relations, and is succinctly expressed in our slogan

"*Throw away the matter and keep the underlying organization.*"

For a sample of the work of Nicolas Rashevsky, I suggest

- *Mathematical Biophysics: Physico-Mathematical Foundations of Biology* (Rashevsky, 1960);
- *Organismic Sets: Some Reflections on the Nature of Life and Society* (Rashevsky, 1972).

### 4.2. Robert Rosen

(1934–1998) continued the line of succession in relational biology, and, among many innovations, instituted the algebraic theory of categories as the natural mathematical metalanguage of the relational theory of life.

Robert Rosen entered Nicolas Rashevsky's Committee on Mathematical Biology at the University of Chicago in the autumn of 1957. Engaged in his work on relational biology, Rosen quickly discovered the (M,R)-systems (metabolism–repair systems; more on these in Section 5 below), and developed some of their extraordinary properties. A happy happenstance was when Rosen connected this relational theory of biological systems to the algebraic theory of categories (founded by Samuel Eilenberg and Saunders Mac Lane in 1945), thus equipping himself with a ready-made mathematical tool. Indeed, Rosen's first published scientific paper was on his (M,R)-systems (Rosen, 1958a), and his *second* paper was on 'The representation of biological systems from the standpoint of the theory of categories' (Rosen, 1958b). His lifetime's work, the quest for the answer to the question "What is life?", culminated in

### 4.3. The Fundamental Theorem of Relational Biology

*A natural system is an organism if and only if it is closed to efficient causation.*

Recall (Definition 2.10) that a natural system is closed to efficient causation if its every efficient cause is (functionally) entailed within the system.

A good overview of the work of Robert Rosen is his trilogy

- *Fundamentals of Measurement and Representation of Natural Systems* (Rosen, 1978);
- *Anticipatory Systems: Philosophical, Mathematical, and Methodological Foundations* (Rosen, 1985);
- *Life Itself: A Comprehensive Inquiry into the Nature, Origin, and Fabrication of Life* (Rosen, 1991);

Supplemented by

- *Essays on Life Itself* (Rosen, 2000).

In Section 5 below I will explore *Life Itself*.

A second motivation for relational biology arises through the employment of *models* for the study of biological processes and their interconnections. Underlying the modelling approach is the concept that *structurally dissimilar systems can nevertheless similarly behave*. Further, *the manifestations of behaviours can also be similar*. Systems with a common model are *analogs* of one another, in which case one can learn about one system by correspondingly studying another. The relational strategy is fundamentally a comparative strategy. Contrariwise, reductionism is intrinsically not comparative, and similar behaviours in different systems can only be individually explained *post hoc*.

### 4.4. Origins

Rashevsky, in 1939, founded the *Bulletin of Mathematical Biophysics* (now the *Bulletin of Mathematical Biology*) after having been taken to task by the editor of a physiology journal because his submitted (and accepted) paper on nervous excitation did not contain "original experimental observations". Rashevsky, as a physicist, began, as was the norm at the time, with an essentially reductionistic view of the relation of biology to chemistry and physics. His successes at modelling specific biological processes, however, instead of reinforcing his reductionism began to gnaw at him in an increasingly something-is-missing sense. As he wrote in Rashevsky (1954):

> There is no record of a successful mathematical theory which would treat the integrated activities of the organism as a whole … this integrated activity of the organism is probably the most essential manifestation of life … These fundamental manifestations of life drop out from all our present theories of mathematical biology … We must look for a principle which connects the different physical phenomena involved and expresses the biological unity of the organism and of the organic world as a whole.

This now-classic 1954 paper ('Topology and Life') is generally acknowledged as the origin of relational biology. Indeed, Rashevsky first discussed therein the 'relational aspects' of biology. By 'relational' he meant an approach that was based on the algebraic, topological organizations of functions, as opposed to one based on the analytic, metric, mechanistic, physicochemical organizations of structures, the latter approach having theretofore dominated his subject of 'mathematical biophysics'.

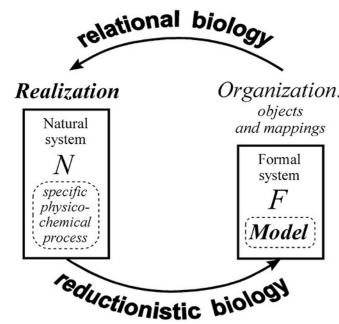
Here is Rashevsky's basic idea: Instead of starting with a mishmash of reductionistic fragments and attempting to find some *a posteriori* way of fitting them together to generate integrated biological behaviour, he might try to represent this integrated behaviour from the outset. Organisms are recognized as such because one can recognize homologies in their behaviours, regardless of the physical structures through which these behaviours are implemented. All organisms seek and ingest food, metabolize it to generate energy, adapt, reproduce, etc. Rashevsky sought to represent the integrated manifestation of these biological functions, common to all organisms, in mathematical terms. Through the basic and ubiquitous manifestation of such functions, organisms could be mapped ('biotopologically') into one another in such a way as to preserve these basic relations, and we could in fact hope to construct a unified theory of organisms in this fashion. In this way, he was led to an abstract topological structure that served as a kind of functional bauplan manifested by any system that might be called an 'organism'. Stated otherwise, one begins with an abstract structure, of which any specific organism constitutes a realization. The manner in which particular organisms relate to (or map onto) the *bauplan* then establishes their relations to one another. Note that a mathematical organization seeking realizations is not yet a model. At this stage, it is decoding without encoding, that is, formally, the top half of the modelling relation (16). This is the essence of *metaphor*. Metaphoric processes decode back into realizations, and induce corresponding causal processes in the realizations, which are then encoded back to the mathematical organization, completing the modelling relation. A metaphor properly decoded and then encoded becomes a model.

Rashevsky coined the term *relational biology* to characterize this qualitative approach, as distinct from the quantitative approach that is *metric biology*. One of its crucial premises is this: Experimenters (e.g., biochemists or molecular biologists) proceed by initially destroying all higher-level biological organization, leaving behind a purely physicochemical system to be studied entirely by physicochemical means. In other words, they proceed by abstracting away all organizational properties, hoping to recapture them in due course by synthetic arguments based on encoded data from their analytic models. The relational approach, on the other hand, proceeds in an exactly converse way; in effect, it initially abstracts away all physicochemical aspects, leaving behind a pure organization to be represented and studied entirely by mathematical means. The detailed physics and chemistry of such a system are to be recaptured later by a decoding process of *realization*.

### 4.5. Premise

The principles of relational biology may thus be considered the operational inverse of reductionistic ideas. Relational biology is mathematical organization seeking realizations, and reductionistic biology is physicochemical process seeking models.

Relational biology decodes; reductionistic biology encodes.



(20)

One must understand that the 'relational' in 'relational biology' is not just an adjective with its common-usage sense of 'having an effect of a connection' (sometimes even misinterpreted as 'relative'). 'Relational' is more importantly used in its mathematical sense that 'a mathematical relation (subset of a product set) exists'.

Operational inverses relational biology and reductionistic biology may be, but it is important to note that we in the former are not antagonistic in any sense towards practitioners of the latter. One does not argue with success. Molecular biology, the poster child of reductionistic biology, is useful and has enjoyed popular success and increased our understanding of life by parts. It is, however, also evident that there are incomparably more aspects of natural systems that the physics of mechanisms is not equipped to explain. It is the overreaching reductionistic claim of *genericity* ("only material-based biology is biology") that is a misrepresentation and a falsehood. The antithesis of relational biology is this presumptuous reductionism (that "biology is chemistry", or exclusive adherence to the bottom-up causal chain, e.g., 'atom → molecule → organelle → cell → tissue → organ → organ-system → organism → community → society').

In relational biology we often propose not one model for a specific biological process, but many entirely different models that share in a common formalism. It is the commonality that makes them all models for a particular biological process which is the true item of interest, and not the detailed properties of any one of them. Thus our interest is not on the details of individual mechanisms (as important as these might be in many contexts), but on qualitative concepts of *behaviour*, which may be variously realized: what needs to be isolated is what these mechanisms share that allows them to be realizations of behaviour.

I emphasize that the relational strategy is our *approach* to our subject, and the *subject* is *mathematical biology*. While 'biology' is the noun and 'mathematical' is the adjective, it is important to note that both biology and mathematics are indispensable ingredients. Their essential integration is due to the fact that what is common between structurally diverse but behaviourally similar systems can best (and often, only) be formulated in abstract terms, in the form of a mathematical system. Natural Law entails the efficacy of mathematics. Beginning with a formal organization, or a metaphor, mathematical analysis then generates a family of elementary mathematical units, or subsystems, in terms of which the abstract behaviours of the formalism may be understood. Such an analytic process decodes back into realizations, and induces corresponding analytic decompositions in the realizations. Formal subsystems thus play the same role for functional organization that the particulates and molecules play for the reductionists' physicochemical structure. The vital (in every sense of the term) characteristics of the relational approach are (i) the analytic subsystems for a particular behaviour need not (and in general do not) correspond to the structural subunits, and (ii) different behaviours require a decomposition of the same system into different sets of analytic subsystems. Relational descriptions can, characteristically, apply to a large class of functionally identical but physically quite distinct systems. For example, the functional term 'active site' is a relational term, and admits only an external, relational description. External descriptions have the property that they can be realized by many distinct systems with different internal descriptions. Thus, in the case of an active site, one may imagine the same site embedded in very different kinds of physical structures. Realizations of 'active site' are thus analogs of one another.

The current exposition is not meant to be an encyclopaedic treatise on relational biology. I invite the enthused reader to further explore the subject in the publications of Rashevsky, Rosen, and me (those already cited above and others). Many biological examples and illustrations of the power of the category-theoretic approach may be discovered therein. I will now proceed towards the special-issue specific foundational topic in the title of this paper.

## 5. Robert Rosen's *Life Itself*

*Life Itself* (Rosen, 1991) is an iconoclastic masterpiece that reaches far beyond the domains of mathematics and biology. Rosen wrote in the Preface of *Essays on Life Itself* (Rosen, 2000):

> … I have been surprised and gratified by the general reaction to *Life Itself*. I have received more correspondence relating to this volume than any other publication of mine, and from a broader spectrum of readers, from orthodox molecular biologists to software developers, linguists, and social scientists. For one reason or another, these correspondents expressed their covert uneasiness with previously presented paradigms; it was a very practical uneasiness, a feeling that their problems were not actually being addressed from those directions. They saw in *Life Itself* a language in which more satisfactory, and more practical, alternatives to current orthodoxies could be expressed. That is exactly what I had hoped.
>
> This correspondence, in fact, reminded me of my fairly extensive travels in eastern Europe years ago. The orthodoxy there, at that time, was Dialectical Materialism, which also promised the solution to everything. Everyone avowed it: It was mandatory to do so. But no one really believed it.

### *5.1. Definition*

An *algorithm* is a process with the following attributes:

i. It terminates after a finite number of steps;
ii. Each step is unambiguously defined;
iii. It has zero or more input data;
iv. It has one or more output data; and
v. It must be effective, which means there must be a Turing-machine equivalent; i.e., the process must be evaluable by a mathematical (Turing) machine.

An algorithm, therefore, is a computation procedure that requires in its application a rigid stepwise mechanical execution of explicitly stated rules. It is presented as a prescription, consisting of a finite number of instructions. It may be applied to any number (including none) of members of a set of possible inputs, where each input is a finite sequence of symbolic expressions. Once the inputs have been specified, the instructions dictate a succession of discrete, simple operations, requiring no recourse to chance and ingenuity. The first operation is applied to the input and transforms it into a new finite sequence of symbolic expressions. This outcome is in turn subjected to a second operation, dictated by the instructions of the algorithm, and so on. After a finite number of steps, the instructions dictate that the process must be discontinued, and some outputs be read off, in some prescribed fashion, from the outcome of the last step.

### *5.2. Definition*

A mapping is *simulable* if it is definable by an algorithm.

Note that the definition of 'algorithm' depends on other computing-theoretic terms, such as *unambiguous* and *evaluable by a mathematical* (*Turing*) *machine* ("*Turing-computable*"). 'Simulable' is also called *computable* and *algorithmically definable*. In the 1930s, there appeared three independent attempts to formalize the notion of computability:

i. *General recursive* (Kurt Gödel, general recursive functions);
ii. $\lambda$*-definable* (Alonzo Church, $\lambda$-calculus);
iii. *Turing-computable* (Alan Turing, *computing machines*).

It has been proven that these three formally defined classes of computable functions coincide. That is, a function is general recursive if and only if it is $\lambda$-definable if and only if it is Turing-computable. Thus it is generally accepted that the concept of *computability* is accurately characterized by these three equivalent processes. For further exploration on these computability-related topics, I refer the reader to Chapter 7 of *Life Itself* and Kercel (2007). For our purpose, we only need to note these three simple

### *5.3. Properties*

*If a mapping is simulable* (*computable*)*, then*

(a) *its corresponding Turing machine halts after a finite number of steps;*
(b) *its corresponding algorithmic process is of finite length; and*
(c) *its corresponding program, which may be considered as a word built of the alphabets of its Turing machine, is of finite length.*

The keyword is *finite*.

A system may be considered a collection of mappings connected by the system's entailment pattern. So one may extend Definition 5.2 and give the

### *5.4. Definition*

A formal system is *simulable* if its entailment pattern and all of its mappings are simulable.

In Chapter 8 of *Life Itself*, Rosen gave the following

### *5.5. Definition*

A natural system *N* is a *mechanism* if and only if all of its models are simulable.

Having *all* of its models simulable is a very stringent condition imposed upon a natural system *N*; and as a consequence, *N* has certain restrictive properties. In Chapter 9 of *Life Itself*, Rosen, using these properties, presented a detailed *reductio ad absurdum* argument that shows certain modes of entailment are *not* available in a mechanism:

### 5.6. Theorem

*There can be no closed path of efficient causation in a mechanism.*

The contrapositive statement of Theorem 5.6 is

### 5.7. Theorem

*If a closed path of efficient causation exists in a natural system N, then N cannot be a mechanism.*

Considering Definition 5.5 of mechanism, this is equivalent to

### 5.8. Theorem

*If a closed path of efficient causation exists for a natural system N, then it has a model that is not simulable.*

Recall from Section 2 that a closed path of efficient causation is a *hierarchical cycle*, a cycle of functional entailments In formal systems, hierarchical cycles are manifested by impredicativities, or the inability to replace these self-referential loops with finite syntactic algorithms. Impredicativities are simply part of the semantic legacy of mathematics as a language, in their expression of transcendental operations. Further elaboration on this concept may be found in abundance in Rosen (2000). The nonsimulable model in Theorem 5.8 contains a hierarchical closed loop that corresponds to the closed path of efficient causation in the natural system being modelled. In other words, it is a formal system with an impredicative loop of inferential entailment. Impredicativity is noncomputable on a Turing machine, which remains the basic conceptual architecture of all computers to-date. Thus one also has:

### 5.9. Theorem

*If an impredicative loop of inferential entailment exists for a formal system, then it is not simulable.*

Rosen gave, in Chapter 19 of Rosen (2000), the following

### 5.10. Definition

A system is *simple* if all of its models are simulable. A system that is not simple, and that accordingly must have a nonsimulable model, is *complex*.

So the definitions of 'simple system' and 'mechanism' are identical. The *complementary* collection of simple systems are complex systems; i. e., a 'complex system' is a system that is *not simple*. Now recall the partition 2.9 of systems, where I have defined $P$ to be the proportion of efficient causes of a system that are involved in a hierarchical cycle. Simple systems are precisely the meagre collection of 'class I' systems in which none of the efficient causes is involved in a hierarchical cycle ($P = 0$), while complex systems are the dense collection of 'class II' systems in which there is at least one hierarchical cycle ($0 < P \le 1$). Because of the impredicative hierarchical cycle, no complex system can be fully functionally modelled by an algorithmic computer.

Rosen's answer to the "What is life?" question is given in Section 10A (aptly entitled "The Answer") of *Life Itself*, that "*a material system is an organism if, and only if, it is closed to efficient causation*". This is, of course, what I have termed The Fundamental Theorem of Relational Biology. An organism is thus a systems of the extreme case $P = 1$, when *all* of its efficient causes are involved in hierarchical cycles. An organism ($P = 1$) is, therefore, a complex system ($0 < P \le 1$). *Complexitas viventia producit.* This inclusion has the following consequence:

### 5.11. Theorem

*A living system must have noncomputable models.*

### 5.12. Rosen's theorem

*An organism must be complex; a complex system may (or may not) be an organism.*

### 5.13. Corollary

*A living system is not a mechanism.*

### 5.14. (M,R)-systems

Robert Rosen introduced (M,R)-systems to the world in 1958, as I have already mentioned, in his very first published scientific paper (Rosen, 1958a). They began as a class of metaphorical, relational paradigms that define cells. The M and R may very well stand for 'metaphorical' and 'relational' in modelling terms, but they are realized as 'metabolism' and 'repair'. The comprehensive reference is Rosen (1972). (M,R)-systems and (M,R)-networks are also topics thoroughly explicated in my three books (Louie, 2009, 2013, & 2017b). The reader may refer to any or all of them for further details. Here I will only provide the minimal

### 5.15. Definition

i. *Metabolism* $M$ = material entailment;
ii. *Repair* $R$ = functional entailment;
iii. A *metabolism-repair network*, i.e., an (M,R)-*network*, is a finite collection of pairs of metabolism and repair components $\langle M_i, R_i \rangle$ such that $R_i \vdash M_i$, connected in a model network;
iv. An (M,R)-*system* is an (M,R)-network that is closed to efficient causation.

In Rosen (1971), he listed three basic kinds of problems arising in the study of (M,R)-systems:

(a) To develop the formal properties of such systems, considered in the abstract, and interpret them in biological terms;
(b) To determine the methods by which the abstract organization which defines the (M,R)-system may be realized in concrete terms;
(c) To determine whether a particular concrete biological system is in fact a realization of an (M,R)-system (i.e. to identify the functional components in a real biological system); this is basically the inverse problem to (*b*).

And he wrote: "Almost all of my published scientific work has arisen from a consideration of these three problems, although this is perhaps not always immediately apparent." This last statement is in fact as true today, when we study Rosen's whole lifetime's work, as it was when he wrote it in 1971.

Rosen argued (in what were in effect concluding statements of his lifetime's work), in Section 10C of *Life Itself*, that "Any material system possessing such a graph [of an (M,R)-system] as a relational model (i.e., which realizes that graph) is accordingly an organism." and, in Chapter 17 of Rosen (2000), that "Making a cell means constructing such a realization. Conversely, I see no grounds for refusing to call such a realization an autonomous life form, whatever its material basis may be." In other words, he proposed the

### 5.16. Postulate of Life

*A natural system is an organism if and only if it realizes an* (M,R)*-system.*

Thus an (M,R)-system is the very model of life, and, conversely, life is the very realization of an (M,R)-system.

A union of interacting (M,R)-systems, or better, their join in the lattice of (M,R)-systems, cf. Sections 2.1 and 7.28 in Louie (2009), is itself an (M,R)-system. A multicellular organism has a life of its own, apart from the fact that the cells that comprise it are alive. Similarly, in some sense an ecosystem of interacting organisms is itself an organism. In particular, a symbiotic union of organisms may itself be considered an organism; cf. Section 11.12 in Louie (2013).

### 5.17. Proofs

Rosen's arguments in *Life Itself* are mathematically sound (although Rosen's presentations are not in the ordinary format of definition-lemma-theorem-proof-corollary that one finds in conventional mathematical texts). But because of the unorthodox heuristic form, people have, since the publication in 1991 of *Life Itself*, contended Rosen's conclusions (as in "Rosen never proved anything!“). But the dispute is over form rather than substance, so it is not surprising that the contentions are mere grumbles, and no logical fallacies in the arguments have ever been found. A common thread running in many of the *anti*-Rosen papers that I have encountered is the following: they simply use definitions of terms different from Rosen's, whence resulting in consequences different from Rosen's, and thereby concluding that Rosen must be wrong! In particular, Rosen's 'simulability' means algorithmic-computability (Turing-computability), that has the *finitude* properties 5.3. Some contrarians have used other definitions of 'computability' (notably not finite or otherwise do not halt) to show that an impredicative process is 'computable', and *therefore* Rosen was wrong in saying that life was noncomputable. This is a very example of the fallacy of moving the goalposts. One of the purposes of my monograph *More Than Life Itself* (Louie, 2009) was to recast Rosen's arguments in *Life Itself* in as rigorously mathematical a footing as possible. Since then, the Rosen-did-not-prove movement has all but disappeared.

### 5.18. Artificium

Note that Rosen's conclusion, a consequence of The Fundamental Theorem of Relational Biology, is that *life is not computable*, *not* that 'artificial life' is impossible. The implication is, rather, however one models life, natural or artificial, one cannot succeed by computation alone. Life is not definable by an algorithm. There is, indeed, practical verification from computer science that attempts in implementation of a hierarchical closed loop leads to deadlock, and is hence forbidden in systems programming (Silberschatz et al., 2018). The reason that the circular hierarchic structure is strictly forbidden in computer science is that it is inherently ambiguous, and its ambiguous structure deadlocks the inherently unambiguous algorithm that attempts to execute it (Knuth, 1973).

That 'artificial life' is not achievable by algorithmic means is not limited to its failure *in silico*. The currently popular kind of 'synthetic biology' — a mechanistic, algorithmic, and by-parts fabrication of life — also will not work, for the same logical impredicativity-is-not-simulable reason. An organism is a complex system; a by-rote synthesis from simple components can only result in a simple system. There is an impermeable barrier between the two classes of systems. Algorithmic life is impossible, because a physical entity that is entirely algorithmic cannot realize a living system that must contain noncomputable hierarchical cycles. For an episode on the synthetic life saga, see Louie (2010).

Eighteenth-century philosophers distinguished the ability to feel (*sentience*) from the ability to think (*sapience*). Sentience is the ability to experience sensations. The internal and subjective component of sense perceptions, arising from stimulation of the senses by phenomena, is known in philosophy of mind as "qualia" (Albertazzi and Louie, 2016; Louie, 2019). Sapience, wisdom, or reason is the ability to think and act using knowledge accumulated through experience, understanding, common sense and insight. One has to be alive before one is sentient, and one has to be sentient before one is sapient. Thus both of these qualities of "mind" are predicated on life. Since life is impredicatve and noncomputable, so are therefore sentience and sapience.

*Intelligence*, however, has a more relaxed definition. While its many definitions vary, a common meaning is the ability to acquire and apply knowledge and skills. Intelligence is often studied in humans but has also been observed in other lifeforms. Intelligence in machines is called *artificial intelligence* (*AI*), a term coined in the 1950s. AI is often used to describe machines (including computers) that *mimic* cognitive functions of the human mind, such as learning and problem solving. The Turing test, developed by Alan Turing in 1950, is a test of a machine's ability to exhibit intelligent behaviour indistinguishable from that of a human being. The premise of the "imitation game" (as the Turing test came to be interpreted) is that if a machine can simulate intelligence then it is intelligent. In other words, artificial intelligence only needs to be a *simulation* of intelligence. (We shall not digress into the strong-AI versus weak-AI issue of the mind–body problem here.)

An entity is not required to be alive before it can be intelligent, therefore, in particular, AI implementation on computing machines does not suffer from the same logical impossibility consequence of The Fundamental Theorem of Relational Biology.

## 6. Attending Church

The vague notion of "effective" began as a characterization of calculability of a class of "intuitively computable" numerical functions. Something can be done "effectively", or a question has an "effective" answer, or that an operation or process is "effective", when there is a corresponding computation procedure. A computation procedure can only use a *finite* amount of information; the description of the procedure, by a list of rules or instructions, must therefore be *finite*. If such a (finite) procedure exists, it is called a *decision procedure* for the given class of questions. The problem of discovering a decision procedure is called the *decision problem* for this class.

### 6.1. Common notion

A computational procedure is *effective* if.

(a) given an input from its domain, it can give the corresponding output by following a finite number of exact unambiguous instructions;
(b) it returns such output (halts) in a finite number of steps;
(c) it uses a finite amount of storage space during computation.

Note that while the word *finite* appears in each of the three properties, there is no prescribed upper bound. There are no limits on the number of arguments, the execution time, or the required storage. For example, in the procedure "choose a number $n \in \mathbb{N}$", since the set $\mathbb{N}$ of natural numbers is infinite, there is no upper bound for the chosen number $n$; but whatever $n \in \mathbb{N}$ happens to be chosen, it must be a finite number. But, of course, when the finiteness is very large, there are resource limitations (technological limits, execution time more than time-to-the-end-of-the-universe, etc.); these *practical* limits, however, do not affect the theoretical 'in-principle' effectiveness of a computational procedure.

### 6.2. Definition

A function for which an effective computational procedure exists is called *effectively calculable*.

Compare Common Notion 6.1 with Definition 5.1. One sees that the requisite properties (i.e., sufficient conditions) are equivalent. An effective method for calculating the values of a function is, then, an algorithm. Indeed, "effectively calculable" and "algorithmically definable" (and hence "Turing-computable") are synonyms. Thus one may say that something can be done "effectively" as a brief way of saying that "there is an algorithm for it". This equivalence leads to

*6.3. Common notion*

*If a function is Turing-computable, then it is effectively calculable* (*or intuitively computable*).

The converse of this statement seems also to be true. Heuristic evidence and other considerations led Alonzo Church in 1936 to propose the following

*6.4. Thesis*

*Every effectively calculable function* (*effectively decidable predicate*) *is general recursive.*

Since general recursiveness and Turing-computability are equivalent (and define "computability"; cf. Section 5), one also has

*6.5. Church–Turing thesis*

*Every effectively calculable function* (*effectively decidable predicate*) *is Turing-computable.*

Together with the converse 6.3, then, one may state the

*6.6. Church's thesis*

*A function is effectively calculable if and only if it is computable.*

Stated otherwise,

*6.7. Church's thesis*

*The concepts of effective calculability and computability are coextensive.*

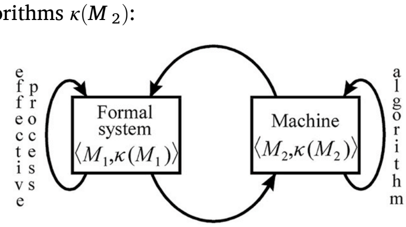
Or, tersely,

*6.8. Church's thesis*

*Effective means computable.*

To say that something is "effective" seems unbearably vague; it is very much like saying that something is "knowable". But Church thought he saw a way to replace that vague, semantic notion of "effectiveness" with an equivalent, precise, purely syntactic one; ironically, by in effect identifying it with syntax itself.

Church's Thesis asserts that any process we would want to call effective can be carried out already by a Turing machine; i.e., purely syntactically. Effective processes can be modelled by a Turing machine; i.e., formal system $\langle M_1, \kappa(M_1)\rangle$ with effectively processes $\kappa(M_1)$ has a 'machine model' $\langle M_2, \kappa(M_2)\rangle$, where $M_2$ is a Turing machine with algorithms $\kappa(M_2)$:




(21)

This is a *thesis* rather than theorem, because it proposes to identify the somewhat vague intuitive concept of "effective" with the concept "computable" that is phrased in exact mathematical terms, and thus the equivalence cannot be proved. The initial intent of Church's Thesis was to characterize vague notions of "effective" calculability and algorithm, and hence programmability in Turing machines. Very strong evidence was adduced by Church, and subsequently by others, in support of the truth of his thesis *in the formal realm*. Church's Thesis equates *effectiveness* with what could be done by iterating rote processes, i.e., with algorithms based entirely on syntax: that is exactly what *computability* means.

*6.9. Realization*

Church's Thesis is most likely true in the formal realm. One may, indeed, *define* a formal process to be effective if and only if it is computable (i.e., if and only if it has a Turing-machine model). Then Church's Thesis becomes *tautologically true*. There are countably many computable functions among uncountably many functions, so by cardinality arguments *most* functions are evidently noncomputable. When Church's Thesis is true, whatever is noncomputable is thereby merely relegated to the category of "ineffective".

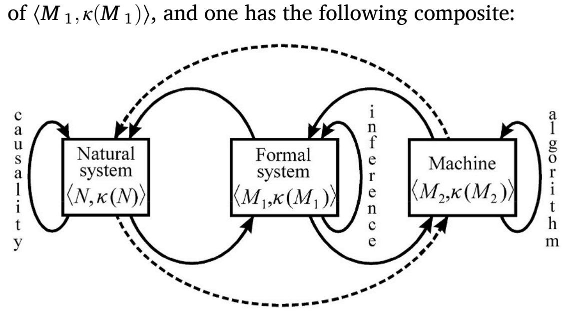
As a conjecture in mathematical logic, Church's Thesis has no real physical content, but it will acquire one when it is decoded into the natural realm. If one attempts to interpret Church's Thesis as a physical proposition, one must give a physical characterization of what one will mean in an operational sense by the term "effective". Nature provides us with a plethora of material processes, governed by causality and not by implication or production rules as in a formal system, which we would want to call "effective". I emphasize that the word "effective" in this context is used in the sense of "actual, existing in fact", "realizable", and "operational"; it is *not* in any volitional or anthropic sense of "efficient", "having a definite or desired effect", or "powerful in effect". Indeed, one has the

*6.10. Common notion*

*Everything that happens in the material world must be considered "effective"*.

For an overview of "physical computability", see Section 4 of Piccinini (2017).

Now suppose the formal system $\langle M_1, \kappa(M_1)\rangle$ is a model of the natural system $\langle N, \ \kappa(N)\rangle$, the prototypical modelling relation being a transition from the material realm to that of mathematics. If *every* causal processes in $\kappa(N)$, whence every inferential process in $\kappa(M_1)$, is effective, one may construct a purely syntactic machine model $\langle M_2, \kappa(M_2)\rangle$ of $\langle M_1, \kappa(M_1)\rangle$, and one has the following composite:




(22)

One may then consider only the outer two systems, and forget about the original model $\langle M_1, \kappa(M_1)\rangle$. The formal system $\langle M_2, \kappa(M_2)\rangle$ is a *machine model* of the natural system $\langle N, \ \kappa(N)\rangle$, and captures the latter's purely syntactic aspects. Given an effective process, one can encode appropriate propositions about the natural system generating it onto a set of input tapes to a Turing machine; and the corresponding output tapes can be decoded so as to perfectly *simulate* the process in question. The encoding and decoding arrows *themselves* (the dashed arrows in (22)) between $\langle N, \ \kappa(N)\rangle$ and $\langle M_2, \kappa(M_2)\rangle$ cannot be described as *effective* in any formal sense, but they compose exclusively with the input and output strings of the Turing machines in $\langle M_2, \kappa(M_2)\rangle$, and these compositions may immediately be identified with the effective processes in $\langle N, \kappa(N)\rangle$.

If Church's Thesis were true in the natural world, then the modelling relation (22) exhausts the causal resources of the system $\langle N, \ \kappa(N)\rangle$. In this context, Church's Thesis means that any causal process can be represented by a corresponding recursive process; i.e., *any causal process can be described by purely syntactic means*. Equivalently, Church's Thesis asserts that all 'information' about material processes, and hence all of Natural Law, can be expressed in purely syntactic terms. If this is true, it of course places severe limitations on what *physics* can be like. Stated otherwise, the truth of Church's Thesis in the material world means the Laws of Nature can themselves be formulated in purely syntactical terms, whereas the falsehood of Church's Thesis in the material world means there are causal processes that possess inherent semantic components that cannot be finitely formalized. Church's Thesis as a physical proposition is thus about, to use a double-negative formulation, the *nonexistence* of *nonsimulable* physical processes. The physical form of Church's Thesis will be true if and only if every mapping in every model of every natural system is simulable. Church's Thesis becomes a direct assertion of the simulability (i.e., the purely syntactic character) of mathematical models of reality (i.e., of systems of *causal* entailments). In short, in accordance of the partitioning of natural systems into simple versus complex in Definition 5.10, one derives the following

*6.11. Consequence*

*The truth of Church's Thesis as a physical proposition implies that every material system is a simple system.*

This contradicts Corollary 5.13: A living system is *not* a simple system. Thus

*6.12.* ⊨ *Conclusion*

*The existence of living systems implies that Church's Thesis is* false *as a physical proposition.*

Let us take stock and see how we arrive at Conclusion 6.12. There are three statements involved:

(S1) Theorem 5.11: *A living system must have noncomputable models*.
(S2) Church's Thesis 6.8: *Effective means computable*.
(S3) Common Notion 6.10: Everything that happens in the material world must be considered "*effective*".

These three statements, logically, can be individually true and pairwise true, but all three cannot be simultaneously true. Note that each statement is a model, and as such, as discussed in Section 4 above, cannot *themselves* be proven, only whether one subjectively agrees.

Statement (S1) is the most rigorously proven, in that it has been mathematically established that a system containing a hierarchical cycle is not Turing-computable. But its truth also depends on The Fundamental Theorem of Relational Biology 4.3, that *a natural system is an organism if and only if it is closed to efficient causation*, which is predicated on modelling a living system as a clef system (and therefore *a fortiori* contains a hierarchical cycle). Equivalently, the Postulate of Life 5.16, that *a natural system is an organism if and only if it realizes an* (M,R)-*system*, is predicated on modelling a living system as an (M,R)-system (which is clef). We in the school of relational biology find the model 'life = clef' convincing, but, of course, others may not concur. The negation of (S1) leads to Consequence 6.11 and allows one to enter the reductionist paradise, in which every material system is a simple system and every natural process is computable.

Statement (S2) is the modelling of effectiveness as computability. The thesis is most likely true in the formal realm, but uncertain in the physical realm. Statement (S3) is the model of every physical process as an effective process. It is a common notion, but what if it is not?

If one accepts the truth of Statement (S1), then either Statement (S2) or Statement (S3) (or both) must be false. So the argument hinges, in effect (pun intended), on the definition of a term (as most arguments are): *effective*. That (S2) is false is Conclusion 6.12. Suppose, on the other hand, one does not subscribe to Common Notion 6.10 (S3), and therefore there are things that happen in the material world that are not "*effective*"; i.e., there exist *ineffective* processes. Just as in the formal realm, one may *define* a physical process to be effective if and only if it is computable. Then Church's Thesis (S2) also becomes tautologically true in the physical realm. The consequence of the falsity of (S3) is the partition of natural processes into effective and ineffective, that a physical process is ineffective if and only if it is noncomputable (with a process that contains a hierarchical cycle as an example).

Before leaving this section, I would like to acknowledge that Robert Rosen first established the connection between relational biology and Church's Thesis in Rosen (1962), then formulated without the language of impredicativity and hierarchical cycles. Rosen subsequently revisited Church a few times. The final outing was Chapter 4 in Rosen (2000): "The Church-Pythagoras Thesis".

## 7. Gödel connections

Kurt Gödel's incompleteness theorems are two theorems in mathematical logic about inherent limitations of axiomatic systems capable of doing a certain amount of elementary arithmetic. The first incompleteness theorem states that no consistent formal system in which theorems can be derived by an algorithmic procedure is complete. Stated otherwise, for any system within which a certain nontrivial amount of elementary arithmetic can be carried out, there will always be statements about the natural numbers that are true but are unprovable within the system. The second incompleteness theorem may be considered a corollary of the first and shows that such a system's own consistency cannot be demonstrated within the system itself. One may note in passing that the 'certain nontrivial amount of elementary arithmetic' requisite in the formal systems are not the same in the two theorems.

The two theorems, proven by Gödel in 1931, are important both in mathematical logic and in the philosophy of mathematics. In essence, the first theorem implies that if something relatively simple like arithmetic already contains such foundational difficulties, then an all-encompassing axiomatic system can never be found that is able to prove all mathematical truths (and nothing but those truths). The contrapositive form of the second theorem says that if an axiomatic system can be proven to be consistent from within itself, then it is inconsistent.

Analogies have often been made to the incompleteness theorems in support of arguments in scholastic pursuits beyond mathematics and logic. Consequently, there has sprung a mini-industry of publications by authors (notably indignant logicians) that have reacted negatively on such 'contaminating' extensions. To be sure, Gödel's incompleteness theorems are about number theory, and statements of their invocation in any other fields are not proven. Just as surely, however, the lessons to be learned from Gödel are more importantly the metaphorical ones.

Indeed, one way to interpret Gödel's Incompleteness Theorem is that it shows the inadequacy of repetitions of rote processes in general. In particular, it shows the inadequacy of the rote metaprocess of adding more rote processes to what is already inside. One knows from Gödel's Theorem that sufficiently rich *formal* systems always contain inferences that cannot be obtained syntactically. More specifically, given any encoding of propositions of the system onto input tapes to Turing machines, there will be propositions of the system which are "true" but will never appear on any output tape. The processes by which the "truth" of such inferences are established are thus *ineffective*; they cannot in principle be simulated by any Turing machine in the given encoding. Changing the encoding will only result in a possibly different set of true-but-unprovable statements, and will not change Gödel's conclusion.

In this connection, the negation of Church's Thesis is also about the inadequacy of repetitions of rote processes in general. If every natural process is effective and effective means computable, then every natural process is computable. In such an impoverished world, every material system is a simple system, and can therefore be synthesized by rote. The cornucopia that is Nature, however, is dense with systems containing hierarchical cycles (of which living systems are examples), and therefore cannot be synthesized by rote. Reductio ad absurdum. QED.

## Declaration of competing interest

None.

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